



Planning For Success Mathematics: Numeration Inquiry Investigations

Operations: Multiplication and Division Number Sense and Numeration

OVERALL EXPECTATIONS

By the end of Grade 4, students will:

- solve problems involving the addition, subtraction, multiplication, and division of single- and multi-digit whole number.

Specific Expectations

Operational Sense

By the end of Grade 4, students will:

- multiply to 9×9 and divide to $81 \div 9$, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting);
- solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies (e.g., 6×8 can be thought of as $5 \times 8 + 1 \times 8$);
- multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule);
- multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., base ten materials or drawings of them, arrays), student-generated algorithms, and standard algorithms;

- divide two-digit whole numbers by one digit whole numbers, using a variety of tools (e.g., concrete materials, drawings) and student-generated algorithms; (e.g., “I can say that of my cubes are 36 means that white, or half of the cubes are white. This 36 and 12 are equal.”);
- use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution (Sample problem: A school is ordering pencils that come in boxes of 100. If there are 9 classes and each class needs about 110 pencils, estimate how many boxes the school should buy).

Proportional Relationships

By the end of Grade 4, students will:

- describe relationships that involve simple whole-number multiplication (e.g., “If you have 2 marbles and I have 6 marbles, I can say that I have three times the number of marbles you have.”);

Patterning and Algebra

OVERALL EXPECTATIONS – GRADE 4

By the end of Grade 4, students will:

- demonstrate an understanding of equality between pairs of expressions, using addition,
- subtraction, and multiplication.

Specific Expectations -Grade 4

Expressions and Equality

By the end of Grade 4, students will:

- determine, through investigation, the inverse relationship between multiplication and division (e.g., since $4 \times 5 = 20$, then $20 \div 5 = 4$; since $35 \div 5 = 7$, then $7 \times 5 = 35$);
- determine the missing number in equations involving multiplication of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (Sample problem: What is the missing number in the equation $\square \times 4 = 24$?);
- identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the commutative property of multiplication to facilitate computation with whole numbers (e.g., “I know that $15 \times 7 \times 2$ equals $15 \times 2 \times 7$. This is easier to multiply in my head because I get $30 \times 7 = 210$.”);
- identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the distributive property of multiplication over addition to facilitate computation with whole numbers (e.g., “I know that 9×52 equals $9 \times 50 + 9 \times 2$. This is easier to calculate in my head because I get $450 + 18 = 468$.”).

Number Sense and Numeration

OVERALL EXPECTATIONS - GRADE 5

By the end of Grade 5, students will:

- solve problems involving the multiplication and division of multi-digit whole numbers, and
- involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies;

Specific Expectations

Quantity Relationships

By the end of Grade 5, students will:

- solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 100 000 (Sample problem: How many boxes hold 100 000 sheets of paper, if one box holds 8 packages of paper, and one package of paper contains 500 sheets of paper?).

Operational Sense

By the end of Grade 5, students will:

- solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies (e.g., use the commutative property: $5 \times 18 \times 2 = 5 \times 2 \times 18$, which gives $10 \times 18 = 180$);
- add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms (e.g., use 10 x 10 grids to add 2.45 and 3.25);
- multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms;
- divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms; use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.

Proportional Relationships

By the end of Grade 5, students will:

- demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation using concrete materials and drawings (*Sample problem: If 2 books cost \$6, how would you calculate the cost of 8*

Patterning and Algebra

OVERALL EXPECTATIONS – GRADE 5

By the end of Grade 5, students will:

- demonstrate, through investigation, an understanding of the use of variables in equations.

Specific Expectations

Patterns and Relationships

Variables, Expressions, and Equations

By the end of Grade 5, students will:

- demonstrate, through investigation, an understanding of variables as changing quantities, given equations with letters or other symbols that describe relationships involving simple rates (e.g., the equations $C = 3 \times n$ and $3 \times n = C$ both represent the relationship between the total cost (C), in dollars, and the number of sandwiches purchased (n), when each sandwich costs \$3);
- demonstrate, through investigation, an understanding of variables as unknown quantities represented by a letter or other symbol (e.g., $12 = 5 + \square$ or $12 = 5 + s$ can be used to represent the following situation: “I have 12 stamps altogether and 5 of them are from Canada. How many are from other countries?”);
- determine the missing number in equations involving addition, subtraction, multiplication, or division and one- or two digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (Sample problem: What is the missing number in the equation $8 = 88 \div \square$?).

Number Sense and Numeration

OVERALL EXPECTATIONS – GRADE 6

By the end of Grade 6, students will:

- solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;

Specific Expectations- Grade 6

Quantity Relationships

By the end of Grade 6, students will:

- solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1 000 000 (*Sample problem:* How would you determine if a person could live to be 1 000 000 hours old? Show your work.);
identify composite numbers and prime numbers, and explain the relationship between them (i.e., any composite number can be factored into prime factors) (e.g., $42 = 2 \times 3 \times 7$).

Operational Sense

By the end of Grade 6, students will:

- use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers (e.g., use the commutative property: $4 \times 16 \times 5 = 4 \times 5 \times 16$, which gives $20 \times 16 = 320$; use the distributive property: $(500 + 15) \div 5 = 500 \div 5 + 15 \div 5$, which gives $100 + 3 = 103$);
- solve problems involving the multiplication and division of whole numbers (four digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms); $12 > 11$;
- add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators;
- multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators (e.g., calculate 4×1.4 using base ten materials; calculate $5.6 \div 4$ using base ten materials);
- multiply whole numbers by 0.1, 0.01, and 0.001 using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule);
- multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies (e.g., “To convert 0.6 m² to square centimetres, I calculated in my head $0.6 \times 10\,000$ and got 6000 cm².”) (*Sample problem:* Use a calculator to help you generalize a rule for multiplying numbers by 10 000.);
- use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution (*Sample problem:* Mori used a calculator to add 7.45 and 2.39. The calculator display showed 31.35. Explain why this result is not reasonable, and suggest where you think Mori made his mistake.);
- explain the need for a standard order for performing operations, by investigating the impact that changing the order has when performing a series of operations (*Sample problem:* Calculate and compare the answers to $3 + 2 \times 5$ using a basic four function calculator and using a scientific calculator.).

Patterning and Algebra

OVERALL EXPECTATIONS – GRADE 6

By the end of Grade 6, students will:

- use variables in simple algebraic expressions and equations to describe relationships.

Specific Expectations

Variables, Expressions, and Equations

By the end of Grade 6, students will:

- demonstrate an understanding of different ways in which variables are used (e.g., variable as an unknown quantity; variable as a changing quantity);
- identify, through investigation, the quantities in an equation that vary and those that remain constant
- solve problems that use two or three symbols or letters as variables to represent different unknown quantities (Sample problem: If $n + l = 15$ and $n + l + s = 19$, what value does the s represent?);
- determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies.

Numeration – Operations of Multiplication and Division

Literature Connections

- Anos Mysterious Multiplying Jar – Mitsumasa, Anno
- Cookies – Jaspersohn
- Annabelle Swift, Kindergartner , Schwartz, Amy(all grades) ie. If A took \$1.08 and milk cost 6 cents, how many milks can she buy
- Beast of Burden from The Man Who Counted: a Collection of Mathematical Adventures, Tahan, Malba (grade 5/6) – ie. 35 camels divided by 3
- Each Orange Had 8 Slices, Giganti, Paul Jr. (all grades) – mental multiplication
- A Remainder of One, Pinczes, Elinor - (grade 5/6) - division

Strategies for Multiplication Facts

Van de Walle book- p.179 - 182

- Doubles – Guide to Effective Instruction Volume 5 – p. 32
- Fives Facts
- Zeros and Ones.
- Nifty Nines
- Helping Facts
- Algorithms – Guide to Effective Instruction Volume 5 – p.37
- Steps for Working through a Multi-Digit Problem- Guide to Effective Instruction Volume 5 – p. 41-43
- Multi-digit Multiplication with Regrouping – Guide to Effective Instruction- Guide to Effective Instruction Volume 5 – p.61-68

Problem Based Multiplication Lessons– Whole Group

- Windows Around the World – Van de Walle - p. 189
- Math From the Ceiling – 50 Problem Solving Lessons – p. 93
- Multiplication and Division – 50 Problem Solving Lessons – p. 155
- Lesson Investigation Multiplication- Guide to Effective Instruction Volume 1 -p. 57

MULTIPLICATION ACTIVITIES/INVESTIGATIONS

All Grades:

- Things That Come in Groups – Van de Walle -page 184
- Packages and Groups – Van de Walle– page 242
- Over and Under – Activity 14.1– Van de Walle – page 256
- High or Low – Activity 14.2 – Van de Walle– page 257
- How Close – Activity 14.8 – Van de Walle– page 265

Grade 4 Investigations:

- Chairs, Chairs and More Chairs – Guide to Effective Instruction- p. 36-40
explore a variety of multiplication strategies
- How Many Fruits? - Guide to Effective Instruction – p. 41,42
arrays and multiplication
- Splitting Arrays – Guides to Effective Instruction – p. 42
- Some More, Some Less – Guide to Effective Instruction – 43
- Whadda Card – Super Source- Base Ten Blocks p. 78
- Cover Up –Super Source- Colour tiles p 38
- Grab Bag Math –Super Source- Snap Cubes – p. 30
- Trains and Boxcars – Super Source-Snap Cubes – p. 78
- The Tile Maker Company –Super Source- Tangrams – p.78
- Shopping for Shapes –Super Source- Tangrams - p.66

Grade 5 Investigations:

- Finding the Cost of a Field Trip – Guide to Effective Instruction – p. 47-53
multiplication strategies
- What Would the Army Look Like? - Guide to Effective Instruction – p. 55
- Exploring the Commutative Property – Guide to Effective Instruction – p. 56
- Fill in the Board – ONAP – p. 24, BLM NS & N, A-2
draw arrays of 2 numbers rolled for multiplication and area
- Packaging Stamps – ONAP – p. 53, BLM NS&N, C2.1 & C2.2
area, arrays and multiplication
- Modelling Multiplication – Super Source – Base Ten Blocks – p. 62
- Nearest Ten – Super Source– Base Ten Blocks – p. 58
- What Happens to the Area – Super Source – Colour Tiles – p. 82

Grade 6 Investigations:

- Shopping for Puppy Food – Guide to Effective Instruction- p. 60-65
multiplication strategies
- Estimating the Cost of Breakfast – Guide to Effective Instruction – p. 66
- Using the Associative Property to Simplify Multiplication – Guide to Effective Instruction-
p. 67
- Halving and Doubling – Guide to Effective Instruction – p. 67-68
- Performance Task: Sharing Strawberries – ONAP – p. 60
multiplication and division
- Multiplication Strategies – ONAP – p. 82
- Modelling Multiplication – Super Source – Base Ten Blocks – p. 62
- Nearest Ten – Super Source– Base Ten Blocks – p. 58
- What Happens to the Area – Super Source – Colour Tiles – p. 82

“Master of multiplication facts and connections between multiplication and division are the key elements in division fact mastery” - page. 183 Van de Walle

PROBLEM BASED DIVISION LESSONS – WHOLE GROUP

- The Game of Left Overs – 50 Problem Solving Lessons – p. 81
dividing with remainders
- A Long Division Activity – 50 Problem Solving Lessons – p. 117
- Multiplication and Division – 50 Problem Solving Lessons – p. 155
- Intramural Dilemmas (grade 4) – Guide to Effective Instruction – p. 35-41, BLM 45
- Family Math Night (grade 5) – Guide to Effective Instruction – p. 48-53
- Gearing Up for a Bike Trip (grade 6) – Guide to Effective Instruction – p. 58-64

DIVISION ACTIVITIES/ INVESTIGATIONS

All Grades:

- How Close and you Get – Van de Walle– Activity 11.18 – page 183
- That's Good Enough – Van de Walle– Activity 14.4 – page 257

Grade 4 Investigations:

- Divide and Draw – Guide to Effective Instruction – p. 41, 42, BLM 45
- Decisions, Decisions – Guide to Effective Instruction – p. 42, 43
- Fair Shares – Guide to Effective Instruction – p. 43, 44
- It's in the Bag – Super Source– Base Ten Blockspage 46
- Even it Up – Super Source-Base Ten Blocks – p 34
- Loose Link– Super Source – Colour Tiles – page 58
- Making Fourths– Super Source – Geoboards – p. 62
- The Cake Problem– Super Source – Geoboard – p. 74
- Boats and Boxes – Super Source– Pattern Blocks – p. 22
- Loose Caboose– Super Source – Snap Cubes – p. 30

Grade 5 Investigations:

- Apples at the Math Fair – Guide to Effective Instruction – p. 53
- Exploring Division – Guide to Effective Instruction – p. 53, 54
- Divisibility Challenge – Guide to Effective Instruction – p. 54
- Striving for Small Remainders – Guide to Effective Instruction – p. 54, 55
- Fair Shares – Super Source– Base Ten Blocks – p.42
- Hundreds of Rectangle– Super Source – Base Ten Blocks. P. 46
- The Square Root Search – Super Source– Base Ten Blocks – p. 82
- Performance Task : Sharing Balloons – ONAP – p.52, NS & N, C1.1 & C1.2

Grade 6 Investigations:

- Exploring a Flexible Division Algorithm – Guide to Effective Instruction – p. 65, 66
- Making Sense of Remainders – Guide to Effective Instruction – p. 66, 67
- Asking Questions – Guide to Effective Instruction – p. 67
- Base Ten Towers – Guide to Effective Instruction – p. 68
- Performance Task: Sharing Strawberries – ONAP – p. 60
multiplication and division

Open Ended Questions

1. You multiply two numbers and the product is almost 400. What could the numbers have been? Explain your thinking.
2. Make up a multiplication/subtraction/addition/ division question where the digits x , x , x , x appear somewhere in the question. * Teacher gives the digits.
3. 4 is a factor of two different numbers. What else could be true about those numbers?
4. You divide two numbers and the answer is 25. What two numbers might you have divided?

5. Write as many equations as you can to describe this array. * Teacher provides the array.
6. Twice as many people came in ahead of David's dad as behind him in the marathon run. If there were 112 participants, what was David's dad's position?
7. Twice as many people came in ahead of David's dad as behind him in the marathon run. What is the possible number of runners?
8. You multiply two numbers together and the product is 40 more than 42×63 . What numbers might you have multiplied?
9. You multiply two numbers together and the product is close to 2600. Both the numbers are greater than 10. What could they be?
10. Are there more multiples of 3 or more multiples of 4 between 1 and 100? How many more?

Online Resources for Student Activities/ Tech Resources

- see various listings - Van de Walle – page 188, 253
- www.multiplication.com
- Math Trek 4,5,6 – program on DDSB teacher tools
- Mathville – program on DDSB teacher tools

Questions to Guide students through the Problem solving model

- What is this investigation about?
- What do you have to do?
- How are you trying to solve the problem?
- What is working well?
- Where are you having difficulties?

Questions to Probe Thinking

- How did you do...?
- How did you know...?
- What happened when you...?
- Why did you....?
- How is this like...?
- What patterns did you see?
- Is there another way to solve this problem?
- How do you know your solution makes sense?