



# Proportional Reasoning for Intermediate Students

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## Which One is Best?



Draw a few rectangles. Choose the one you think is the most visually pleasing. Compare your favourite with others in your group.

As a group choose the most pleasing? Did you have a similar rectangle?

Find the ratio of the length:width in fraction form.

Put this fraction into a decimal. Compare the decimals around the room. Record.

Create a golden rectangle.

# Golden Ratio



Now look at the following sequence of numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89...

The following number is the sum of the previous two. This is Fibonacci's sequence.

Compare the ratios between the numbers. How do they change?

Now do the following ratios on a calculator and give answers in decimal numbers.

$$1/1 =$$

$$1/2 =$$

$$3/2 =$$

$$5/3 =$$

$$8/5 =$$

$$13/8 =$$

$$21/13 =$$

$$34/21 =$$

$$34/21 =$$

$$55/34 =$$

$$89/55 =$$

$$144/89 =$$

$$233/144 =$$

$$377/233 =$$

$$610/377 =$$

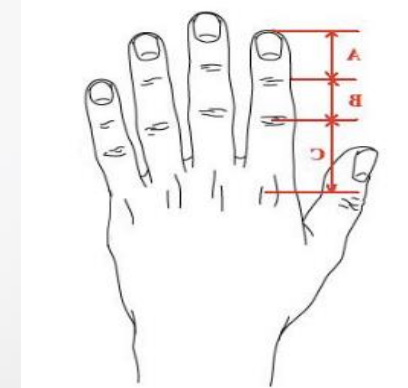
$$987/610 =$$

What pattern do you see? Graph your results.

# Ratios in the Human Body

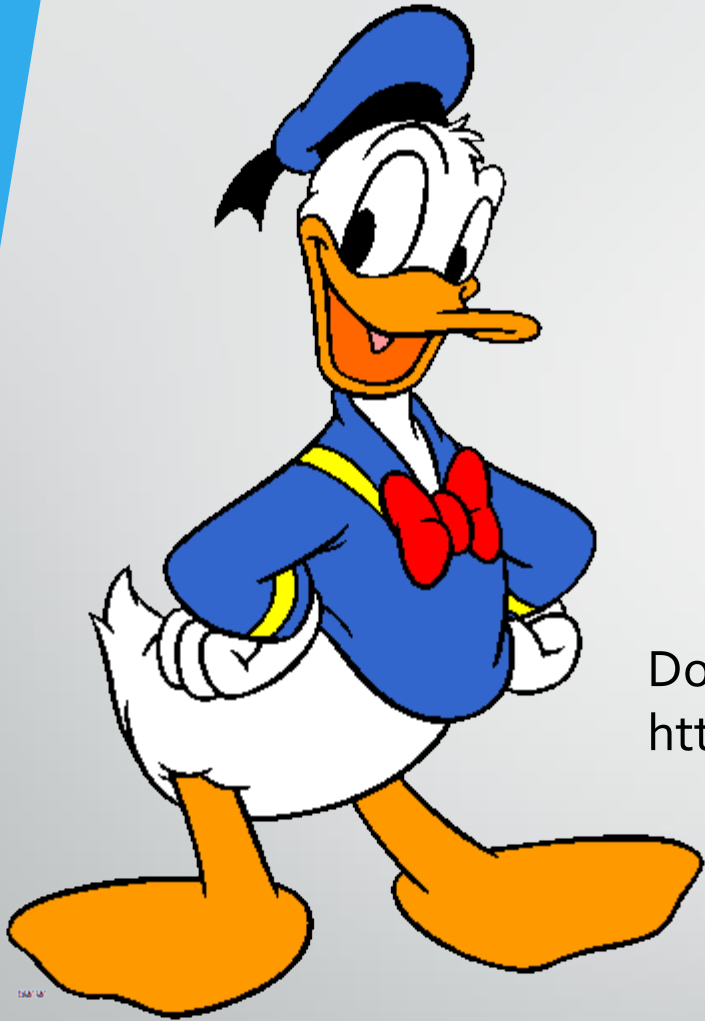
## Part 1: Ratio in the Human Body

- Measure and record the following:
- Distance from the ground to your belly button \_\_\_\_\_
- Distance from your belly button to the top of your head \_\_\_\_\_
- Distance from the ground to your knees \_\_\_\_\_
- Distances A, B and C (see hand)
- Length of your hand \_\_\_\_\_
- Distance from your wrist to your elbow \_\_\_\_\_



## Part 2: Now calculate the following ratios:

- Ratio 1: Distance from the ground to your belly button / Distance from your belly button to the top of your head
- Ratio 2: Distance from the ground to your belly button / Distance from the ground to your knees
- Ratio 3: Distance C / Distance B
- Ratio 4: Distance B / Distance A
- Ratio 5: Distance from your wrist to your elbow / Length of your hand



## Video Clip

Donald Duck in Mathematic Land - 8:39  
<https://www.youtube.com/watch?v=navokVa66xk>



# RESEARCH

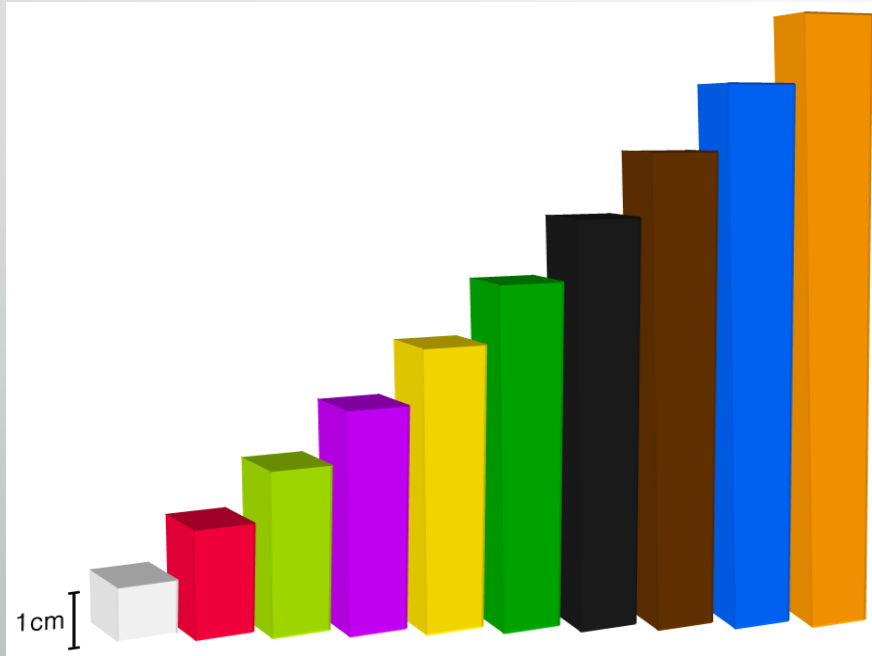
- Proportional reasoning goes well beyond the notion of setting up a proportion to solve a problem - it is a way of reasoning about multiplicative situations. Proportional reasoning, like equivalence, is considered a unifying theme in mathematics. Proportionality is a complex topic. It is estimated that over half of the adult population does not reason proportionally. -S. Lamon (1996)
- Failure to develop in this area by early to middle adolescence precludes study in a variety of disciplines requiring quantitative thinking and understanding, including algebra, geometry, some aspects of biology, chemistry, and physics. (Hoffer and Hoffer, 1988)...Proportionality is clearly not "fluff" mathematics. It is broad, deep, and significant enough to unify many concepts into a major theme of middle grades mathematics. -Lanius and Williams. MTMS, April 2003.

# Proportional Reasoning: A Unifying Theme in Mathematics

## Geometry / Measurement

- Measurement:
  - Ratio of Right Prism Perimeter : Diameter      Pi   Ratio of C: D
  - If you have a Cheerios box and you half each of the dimensions, what will happen to the volume? Surface area? Why?
  - How does doubling the diagonal of a square affect its perimeter and area?
- Geometry
  - Sarah claims that when two triangles have one angle the same size, the triangles have proportional sides. Do you agree? Why or why not?
  - How can you create a similar rectangle by reducing or enlarging its size?
  - How can you use graph paper to enlarge a picture?
  - Similar Rectangles

# Manipulatives





# What are the Different Types of Ratios?

1. Part to Part Ratios: 3 boys to 4 girls (7 in whole) Part to part ratios are not fractions.
2. Part to Whole Ratios: 3 boys to 7 total
3. Ratios as Quotients: 4 kiwis for \$2 or 50 cents each. This is the unit ratio.
4. Ratios as Rates: Rates involve 2 different units and how they relate to each other.

# Which Dog Gained More Weight?



5 kilograms



8 kilograms

Dog #1



3 kilograms



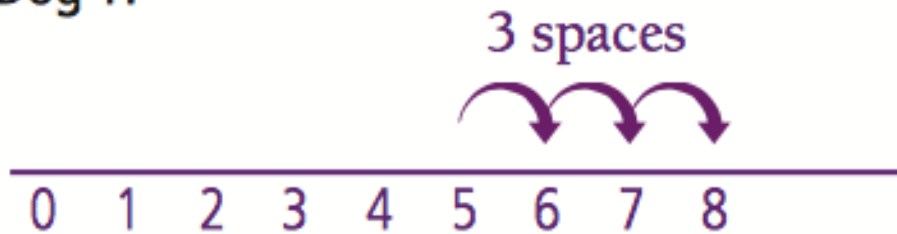
6 kilograms

Dog #2

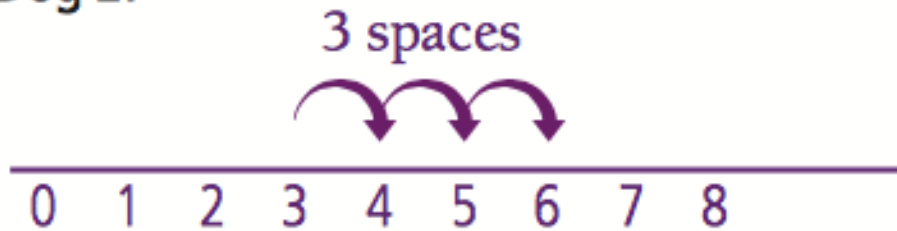
*Here is student thinking that is absolute:*

*The first dog grew by 3kg. The second dog grew by 3 kg. They grew the same amount."*

**Dog 1:**



**Dog 2:**



*Here is student thinking that is relative:*

*The second dog grew more since he doubled his previous weight, unlike the first dog who would have needed to be 10 kg to grow by the same relative amount.*

**Dog 1**



**Less than  
doubled weight**

**Dog 2**



**Doubled  
weight**

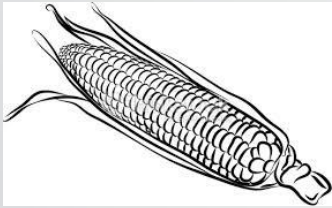
# Is It or Isn't It Proportional?

- Mr. Miller's 25 students are asked if they are basketball fans (yes or no). Of these students, 20 say yes and 5 say no.
  - Describe as many relations as you can about this data.
  - Find at least one additive and 2 proportional relationships.
- Students might describe:
  - There are 15 more fans than nonfans. (additive)
  - There are 4 times as many fans as nonfans (proportional)
  - For every 4 students who like basketball, there is 1 who doesn't (proportional)



# Proportional Reasoning or Not

- Janet and Jeanette are walking to school, each at the same rate. Jeanette started first. When Jeanette has walked 6 blocks, Janet has walked 2 blocks. How far will Janet have walked when Jeanette is at 12 blocks?

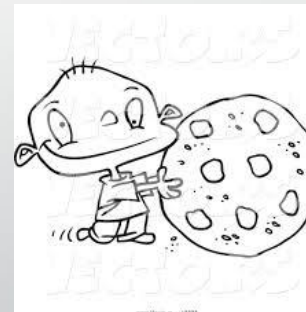


- Lisa and Linda are planting corn on the same farm. Linda plants 4 rows and Lisa plants 6 rows. If Linda's corn is ready to pick in 8 weeks, how many weeks will it take for Lisa's corn to be ready?
- Kendra and Kevin are using the same recipe to bake cookies. Kendra will make 6 dozen cookies, and Kevin will make 3 dozen cookies. If Kevin uses 6 ounces of chocolate chips, how many ounces will Kendra need?

## Identify the type of problem and explain why.

Consider additive, multiplicative or constant.

Make up your own problem for each type.



# Strategies

Proportional Reasoning involves comparing 2 ratios as well as identifying equivalent ratios. There are many methods of solving problems:

- Unit ratios (Unitizing) – Figure out the unit ratio
- Scaling up or down – (If 3 dozen cost 24.99, what would 6 dozen cost? You could use unit rate, but it would be easier to multiple the cost by 2. This is a build up or scale approach)
- Scale factors – Comparing the scale and using that factor to calculate the answer.
- Ratio tables
- Double line comparison
- Graphs
- Cross products



All these methods are useful in particular situations and all should be understood. The first three are the most intuitive, therefore the students might be able to discover them independently. These problems will allow them to explore some of these strategies.



# Strings - Unit Ratios

Find 2 unit ratios and 2 scale up and 2 scale down ratios.

9 km in 3 hours

3 km in 1 hour

3 km in 60 minutes

1 km in 20 minutes

$\frac{1}{2}$  km in 10 minutes

1000 m in 20 minutes

50 m in 1 minute

# Per Cents – Unit Ratios

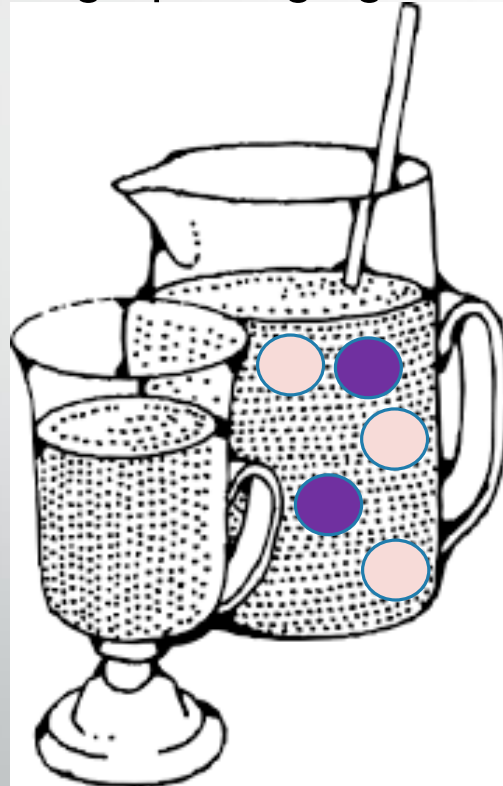
- Estimate 20 % off of \$345
- 10 % of 345 is about \$35
- 20% of 345 is about 70
  
- Estimate 15% tip on \$79
- 10% of \$79 is about \$8
- 5% of \$79 is about \$4
- Tip should be about \$12



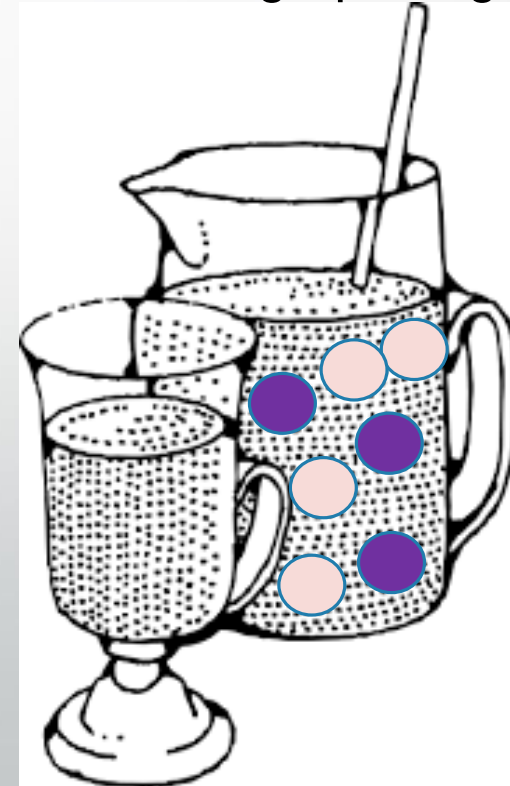
# How Will They Taste?

Will they taste the same?

Pitcher #1: 2 grape: 3 ginger ale



Pitcher #2 3 grape: 4 ginger ale



# Who Had More Pizza?



Two groups of students, McDonald's Gr. 7/8 and the Sampson's Gr. 8 are having pizza parties. Mme McDonald ordered enough so that every 3 students will have 2 pizzas. Mme Sampson orders 3 pizzas for every 5 students.

Assuming the same size of pizza, which groups of students had more pizza to eat?

# Field Trip

Mrs. Middleton's split grade travelled on a field trip. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch the subs were cut and shared as follows:

- The first group had 4 people and shared 3 subs equally.
- The second group had 5 people and shared 4 subs equally.
- The third group had 8 people and shared 7 subs equally.
- The last group had 5 people and shared 3 subs equally.



When they returned from the field trip, the Nico began to argue that the distribution of the sandwiches had not been fair, that some got more to eat than the others. Was he right? Or did everyone get the same amount?

Group #1



- Group #2



# The World Most Successful Dieter



The World Most Successful Dieter is to win \$100 000.

Max, Moe and Minnie are each on a diet and have recorded their weight in kilograms at the start of their diet and at 2 week intervals. After four weeks, which person is the most successful dieter?

The task is to make three different arguments – one that would favour each of the 3 dieters?

Choose your champion and be prepared to defend your choice.

Week	Max (kg)	Moe (kg)	Minnie (kg)
0	95	71	51
2	91	69	49
4	85	67	45

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  - Cross products
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# Ratio Tables



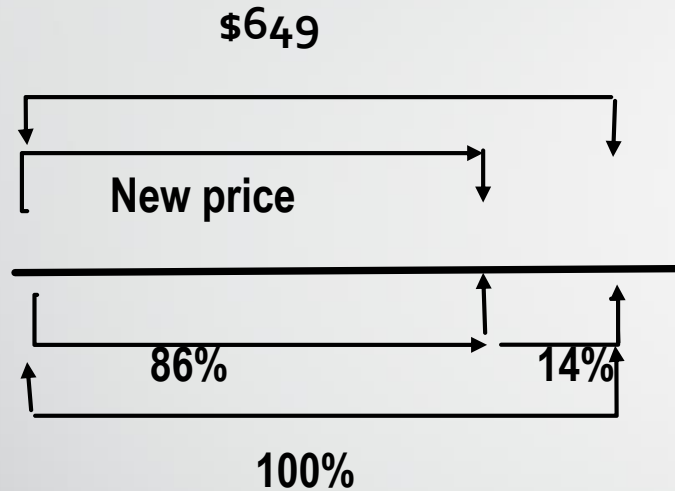
Daniel and Nick are running a race. Daniel runs 3 km every 17 minutes. Nick runs 5 km every 29 minutes. Who runs faster?

	17 minutes	34 minutes	51 minutes	68 minutes	85 km
Daniel	3 km	6 km	9 km	12 km	15 km

	29 minutes	58 minutes	87 minutes	116 min.	145 min.
Nick	5 km	10 km	15 km	20 km	25 km

# Double Line Comparisons / Cross Products

Arsheeya purchased a new cell phone for 14% off. The original \$649. What was the new price?



Use double line chart to set up ratio.

$$\frac{\text{New price of phone}}{\text{Original phone price}} = \frac{86\%}{100\%}$$







# Proportional Reasoning In Real Life

"The probability of being watched is directly proportional to the stupidity of your act."

<https://www.youtube.com/watch?v=G8DNGh126oM>