All these problems were inspired by questions that appeared on a Canadian Team Math Contests, for high school students created by the Centre for Education in Mathematics and Computing at the University of Waterloo. For more contests and other resources visit their website: `http://cemc.uwaterloo.ca`

1. Three siblings share a box of chocolates.

   (a) Andrew eats $\frac{1}{3}$ of the total number of chocolates and Betty eats $\frac{3}{8}$ of the total number of chocolates. Cecily eats the remaining chocolates in the box. Who ate the most chocolates and who ate the least?

   (b) Andrew eats $\frac{1}{3}$ of the total number of chocolates and Betty eats $\frac{3}{8}$ of the *remaining* chocolates. Cecily eats the remaining chocolates in the box. Who ate the most chocolates and who ate the least?

   (c) Create another situation where the person who ate the most and least is different from parts (a) and (b).

2. Evaluate each of the following:

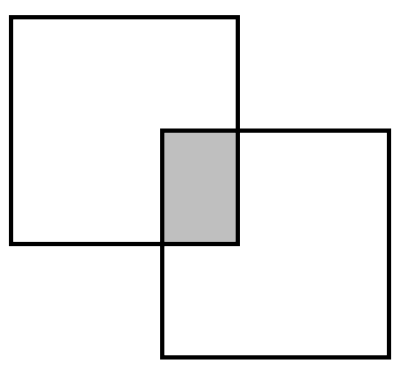
   (a) $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}}$

   (b) $\dfrac{\frac{1}{2}+\frac{1}{5}}{\frac{1}{2}\times\frac{1}{5}}$

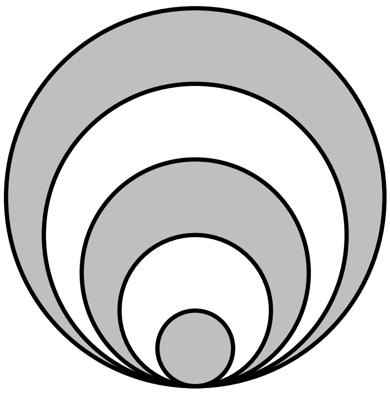
   (c) $\dfrac{\frac{1}{3}+\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}}$

   (d) Discover a rule for evaluating problems of this type. Justify your answer.

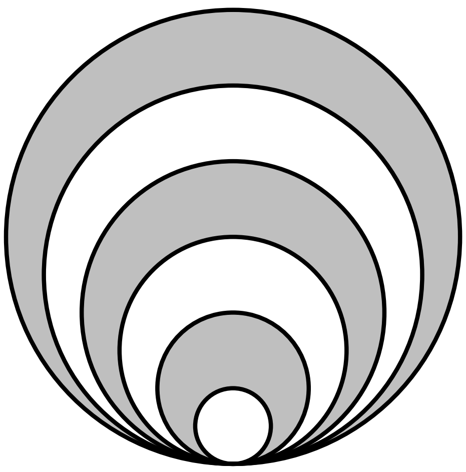
3. A figure is formed by overlapping two squares. The squares each have sides 6 cm and the rectangular overlap is a 2 cm by 3 cm, as shown in the diagram below. What fraction of the figure is shaded?



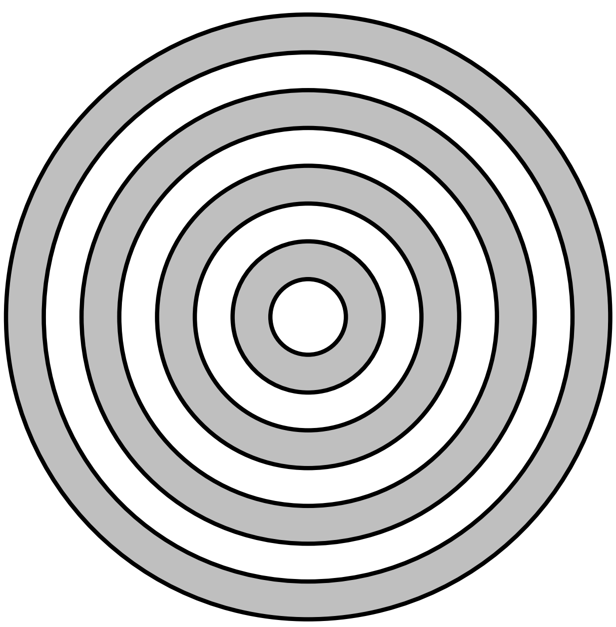
4. Determine the fraction of the areas of the figures below that is shaded. In each case the radius of the smallest circle is 1 cm and each larger circle has radius 1 cm larger than the one before it.



(a)



(b)



(c)

Solutions/comments/suggestions

1. This item was inspired by relay question #2(b) from the 2014 Canadian Team Math Contest.

   Three siblings share a box of chocolates.

   (a) *Andrew eats $\frac{1}{3}$ of the total number of chocolates and Betty eats $\frac{3}{8}$ of the total number of chocolates. Cecily eats the remaining chocolates in the box. Who ate the most chocolates and who ate the least?*

   Andrew eats $\frac{1}{3} = \frac{8}{24}$ of the chocolates while Betty eats $\frac{3}{8} = \frac{9}{24}$ of the chocolates. Thus Cecily eats

$$\begin{aligned} 1 - \left(\frac{1}{3} + \frac{3}{8}\right) &= 1 - \left(\frac{8}{24} + \frac{9}{24}\right) \\ &= \frac{24}{24} - \frac{17}{24} \\ &= \frac{7}{24} \end{aligned}$$

   of the chocolates.

   $\therefore$ Betty eats the most chocolates ($\frac{9}{24}$ of the box) while Cecily eats the least ($\frac{7}{24}$ of the box). Andrew eats more that Cecily but less than Betty ($\frac{8}{24}$ of the box).

   (b) *Andrew eats $\frac{1}{3}$ of the total number of chocolates and Betty eats $\frac{3}{8}$ of the* remaining *chocolates. Cecily eats the remaining chocolates in the box. Who ate the most chocolates and who ate the least?*

   Andrew eats $\frac{1}{3} = \frac{8}{24}$ of the chocolates, so there is $1 - \frac{1}{3} = \frac{2}{3}$ of the box remaining. Then Betty eats $\frac{3}{8}$ of the $\frac{2}{3}$ of the chocolates or

$$\frac{3}{8} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$$

   of the chocolates. Thus Cecily eats

$$\begin{aligned} 1 - \left(\frac{1}{3} + \frac{1}{4}\right) &= 1 - \left(\frac{8}{24} + \frac{6}{24}\right) \\ &= \frac{24}{24} - \frac{14}{24} \\ &= \frac{10}{24} = \frac{5}{12} \end{aligned}$$

   of the chocolates.

   $\therefore$ Cecily eats the most chocolates ($\frac{10}{24}$ of the box) while Betty eats the least ($\frac{6}{24}$ of the box). Andrew eats more that Betty but less than Cecily ($\frac{8}{24}$ of the box).

(c) *Create another situation where the person who ate the most and least is different from parts (a) and (b).*

This is the place where the student's creativity can come into play. It is suggested that students have manipulatives to help them create their new problem.

2. This item was inspired by relay question #2(a) from the 2013 Canadian Team Math Contest.

Evaluate each of the following:

(a) $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}}$

$$\begin{aligned}\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}} &= \frac{\frac{3}{6}+\frac{2}{6}}{\frac{1}{2}\times\frac{1}{3}}\\ &= \frac{\frac{5}{6}}{\frac{1}{6}}\\ &= \frac{5}{6}\times\frac{6}{1}\\ &= \frac{30}{6}=5\end{aligned}$$

(b) $\dfrac{\frac{1}{2}+\frac{1}{5}}{\frac{1}{2}\times\frac{1}{5}}$

$$\begin{aligned}\frac{\frac{1}{2}+\frac{1}{5}}{\frac{1}{2}\times\frac{1}{5}} &= \frac{\frac{5}{10}+\frac{2}{10}}{\frac{1}{2}\times\frac{1}{5}}\\ &= \frac{\frac{7}{10}}{\frac{1}{10}}\\ &= \frac{7}{10}\times\frac{10}{1}\\ &= \frac{70}{10}=7\end{aligned}$$

(c) $\dfrac{\frac{1}{3}+\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}}$

$$\begin{aligned}\frac{\frac{1}{3}+\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}} &= \frac{\frac{5}{15}+\frac{3}{15}}{\frac{1}{3}\times\frac{1}{5}}\\ &= \frac{\frac{8}{15}}{\frac{1}{15}}\\ &= \frac{8}{15}\times\frac{15}{1}\\ &= \frac{90}{15}=8\end{aligned}$$

(d) *Discover a rule for evaluating problems of this type. Justify your answer.*

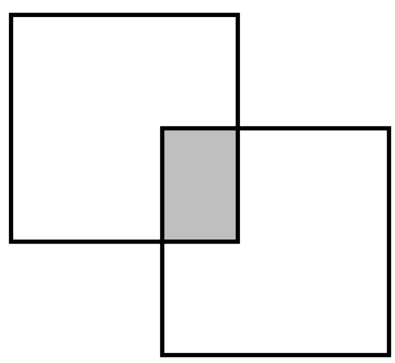
It seems from the problems that the answer is just the sum of the denominators of the fractions involved. So if the denominators were a and b we would get

$$\begin{aligned}\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}\times\frac{1}{b}} &= \frac{\frac{b}{ab}+\frac{a}{ab}}{\frac{1}{a}\times\frac{1}{b}} \\ &= \frac{\frac{b+a}{ab}}{\frac{1}{ab}} \\ &= \frac{b+a}{ab}\times\frac{ab}{1} \\ &= \frac{ab(b+a)}{ab} = b+a\end{aligned}$$

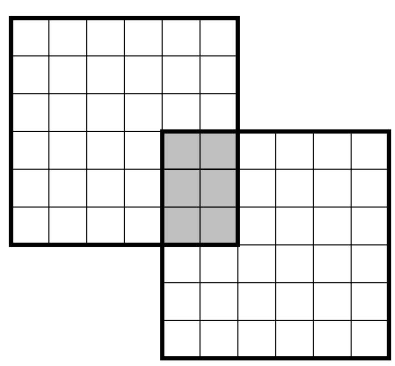
so our conjecture is correct.

3. This item was inspired by relay question #3(c) from the 2013 Canadian Team Math Contest.

A figure is formed by overlapping two squares. The squares each have sides 6 cm and the rectangular overlap is a 2 cm by 3 cm, as shown in the diagram below. What fraction of the figure is shaded?

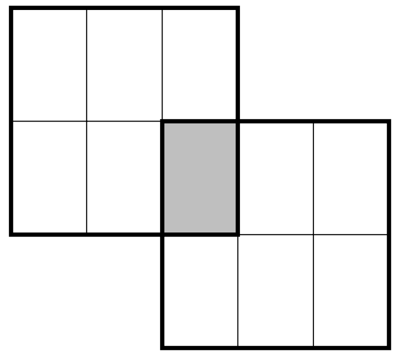


<u>Solution 1:</u> Break the figure up into 1 cm by 1 cm squares and you get



You can see that 6 of the 66 squares are shaded, so the fraction is $\dfrac{6}{66}=\dfrac{1}{11}$.

Solution 2: Break the figure up into 2 cm by 3 cm rectangles and you get



You can see that 1 of the 11 squares is shaded, so the fraction is $\frac{1}{11}$.

Solution 2: The shaded region is $\frac{2}{6} = \frac{1}{3}$ of the square's side long and $\frac{3}{6} = \frac{1}{2}$ of the square's side wide. So the area of the rectangle is

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

of the area of the square. Since both squares overlap the same region, the area of the whole figure is $2 - \frac{1}{6} = \frac{11}{6}$ the area of one square. So the fraction shaded is

$$\frac{\frac{1}{6}}{\frac{11}{6}} = \frac{1}{11}.$$

4. This item was inspired by question #1 from the 2005 Fryer Contest.