All these problems appeared on a Fryer contest, for grade 9 students created by the Centre for Education in Mathematics and Computing at the University of Waterloo. For more contests and other resources visit their website: http://cemc.uwaterloo.ca

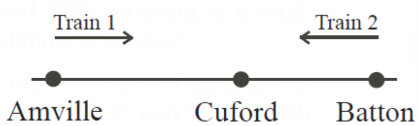
1. (a) In Carrotford, candidate A ran for mayor and received 1008 votes out of a total of 5600 votes. What percentage of all votes did candidate A receive?

   (b) In Beetland, exactly three candidates, B, C and D, ran for mayor. Candidate B won the election by receiving $\frac{3}{5}$ of all votes, while candidates C and D tied with the same number of votes. What percentage of all votes did candidate C receive?

   (c) In Cabbagetown, exactly two candidates, E and F, ran for mayor and 6000 votes were cast. At 10:00 p.m., only 90% of these votes had been counted. Candidate E received 53% of those votes. How many more votes had been counted for candidate E than for candidate F at 10:00 p.m.?

   (d) In Peaville, exactly three candidates, G, H and J, ran for mayor. When all of the votes were counted, G had received 2000 votes, H had received 40% of the votes, and J had received 35% of the votes. How many votes did candidate H receive?

3. Train 1 is travelling from Amville to Batton at a constant speed.
   Train 2 is travelling from Batton to Amville at a constant speed.



   (a) Train 1 travels at 60 km/h and travels $\frac{2}{3}$ of the distance to Batton in 9 hours. Determine the distance from Amville to Batton.

   (b) Train 2 travels $\frac{2}{3}$ of the distance to Amville in 6 hours. How fast is the train going?

   (c) Train 2 started its trip $3\frac{1}{2}$ hours after Train 1 started its trip. Both trains arrived at Cuford at 9:00 p.m. What time did Train 1 leave Amville?

2. If a team won 13 games and lost 7 games, its *winning percentage* was $\frac{13}{13+7} \times 100\% = 65\%$, because it won 13 of its 20 games.

   (a) The Sharks played 10 games and won 8 of these.
   Then they played 5 more games and won 1 of these.
   What was their final winning percentage? Show the steps that you took to find your answer.

   (b) The Emus won 4 of their first 10 games.
   The team played x more games and won all of these.
   Their final winning percentage was 70%.
   How many games did they play in total? Show the steps that you took to find your answer.

   (c) The Pink Devils started out the season with 7 wins and 3 losses.
   They lost all of their games for the rest of the season.
   Was there a point during the season when they had won exactly $\frac{2}{7}$ of their games? Explain why or why not.

4. Points B, C, and D lie on a line segment AE, as shown.

The line segment AE has 4 *basic sub-segments* AB, BC, CD, and DE, and 10 *sub-segments*: AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE.
The *super-sum* of AE is the sum of the lengths of all of its sub-segments.

(a) If $AB = 3$, $BC = 6$, $CD = 9$, and $DE = 7$, determine the lengths of the 10 sub-segments, and calculate the super-sum of AE.

(b) Explain why it is impossible for the line segment AE to have 10 sub-segments of lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

(c) When the super-sum of a new line segment AJ with 9 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ is calculated, the answer is 45.
Determine the super-sum of a line segment AP with 15 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{15}$.

1. Lloyd is practising his arithmetic by taking the reciprocal of a number and by adding 1 to a number.
Taking the reciprocal of a number is denoted by $\xrightarrow{R}$ and adding 1 is denoted by $\xrightarrow{A}$.
Here is an example of Lloyd's work, starting with an input of 2:

$$2 \xrightarrow{R} \frac{1}{2} \xrightarrow{A} \frac{3}{2} \xrightarrow{R} \frac{2}{3} \xrightarrow{A} \frac{5}{3} \xrightarrow{R} \frac{3}{5}$$

(a) Using an input of 3, fill in the five blanks below:

$$3 \xrightarrow{R} \_\_\_ \xrightarrow{A} \_\_\_ \xrightarrow{R} \_\_\_ \xrightarrow{A} \_\_\_ \xrightarrow{R} \_\_\_$$

(b) Using an input of x, use the same operations and fill in the five blanks below:

$$x \xrightarrow{R} \_\_\_ \xrightarrow{A} \_\_\_ \xrightarrow{R} \_\_\_ \xrightarrow{A} \_\_\_ \xrightarrow{R} \_\_\_$$

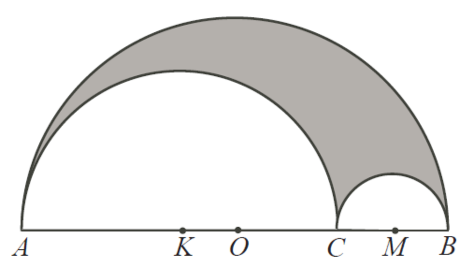
(c) Using the five steps from (b), what input should you begin with to get a final result of $\frac{14}{27}$?
Justify your answer.

3. In the diagram, K, O and M are the centres of the three semi-circles. Also, $OC = 32$ and $CB = 36$.

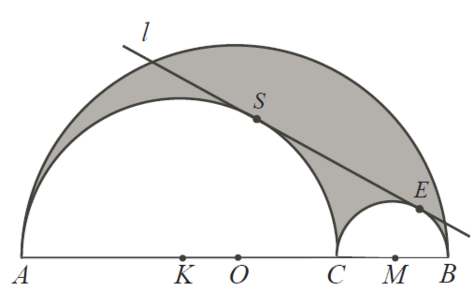
(a) What is the length of AC?

(b) What is the area of the semi-circle with centre K?

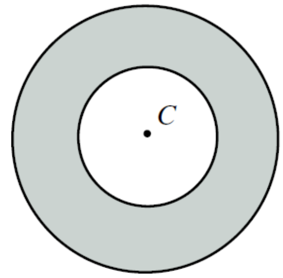
(c) What is the area of the shaded region?



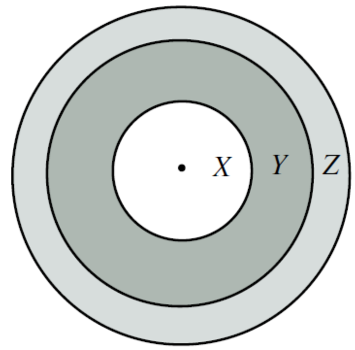
(d) Line l is drawn to touch the smaller semi-circles at points S and E so that KS and ME are both perpendicular to l. Determine the area of quadrilateral $KSEM$.



1. (a) Two circles have the same centre C. (Circles which have the same centre are called *concentric*.) The larger circle has radius 10 and the smaller circle has radius 6. Determine the area of the ring between these two circles.



(b) In the diagram, the three concentric circles have radii of 4, 6 and 7. Which of the three regions X, Y or Z has the largest area? Explain how you got your answer.



(c) Three concentric circles are shown. The two largest circles have radii of 12 and 13. If the area of the ring between the two largest circles equals the area of the smallest circle, determine the radius of the smallest circle. Explain how you got your answer.