Student Misconceptions Aligned with the Fractions Learning Pathway

Uncovering Misconceptions Through Student Clinical Interviews

Facilitated by:
Jennifer Fannin, Michelle Bailey, Nicole Brown & Christine Haladay
**OTF Grant Learning Goals**

1. Review current literature on fraction concepts
2. Make correlations between Fractions Learning Pathway and Ontario Curriculum
3. Create structured interview questions based on Fractions Learning Pathway and Ontario Curriculum to determine student misconceptions
4. Interview students to better understand trends in their misconceptions and areas to focus instruction
5. Create Number Talks and structured lessons to close gaps and clarify misconceptions related to fractions
Portage Trail Community School is an inner city school in Toronto. It is currently classified as a “math increased” school receiving targeted coaching support in mathematics. Based on large gaps in conceptual understanding as evidenced in CAT 4 and EQAO results staff wanted to undergo focused research on student misconceptions intending to use this information to carefully focus instruction.
### Fractional Models

<table>
<thead>
<tr>
<th>Linear Models/Number line (Continuous)</th>
<th>Area Model (Continuous)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a linear model, a length is divided into fractional parts.</td>
<td>In an area (region) model, one shape or object represents the whole. The whole is divided into fractional regions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume Model</th>
<th>Set Model (Discrete)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a volume model, a three-dimensional figure represents the whole. The whole is divided into fractional regions that are occupied by space within the figure.</td>
<td>In a set model, a collection of objects represents the whole amount. Subsets of the whole make up the fractional parts.</td>
</tr>
</tbody>
</table>
Fractions Learning Pathways

Please Note:
- Mixed, improper and proper fractions should be interspersed throughout fractions teaching and learning so that the students build flexibility with these early.
- "Models" include linear, area, volume, and set representations.

Created by Dr. Cathy Bruce, Tara Plynn and Shelley Yearley.
Fractions Learning Pathways are inspired by Dr. Jon Star's work, based on international and Ontario classroom research, and informed by feedback from classroom teachers and student thinking.
## Interview Questions: Unit Fractions

### Unit Fractions

**Unit A**

Use proportional reasoning to make reasonable estimates

### Grade 2

- determine, through investigation using concrete materials, the relationship between the number of fractional parts of a whole and the size of the fractional parts

### Grade 3

- divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation

### Which represents a half?

**Materials to be provided:**

- \( \frac{1}{2} \) (standard notation)
- Coloured tiles blue & green separated \( \frac{1}{2} \) (area)
- Measuring cup \( \frac{1}{2} \) (volume)
- 2 quarters & 2 blank spaces (set)
- Hundreds chart 50/100 (area)
- Box of crayons/markers with one removed (set)
- Cuisenaire rod (brown & two green) (linear)
Unit Fractions Misconception Trends

Identify the fractions that show $\frac{1}{2}$.

Unit Fractions

Unit A
Use proportional reasoning to make reasonable estimates
Unit Fractions Misconception Trends

Identify the fractions that show \( \frac{1}{2} \).

### Unit Fractions

**Unit A**

Use proportional reasoning to make reasonable estimates

### Misconception Trends Unit A

- Does not recognize a fraction when fractional parts are separated by space
  
  “This is not 2/6”

- Does not recognize a fraction when fractional parts are not all the same shape
  
  “This is not \( \frac{1}{4} \)”

- Misidentifies fractions of a set due to confusion with area model
  
  “This is not \( \frac{1}{4} \). The star is bigger than the circles”
## Interview Questions: Unit Fractions

<table>
<thead>
<tr>
<th>Unit Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit E</strong></td>
</tr>
<tr>
<td>Use unit fractions to compose and decompose fractions with models and symbols.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade 4</th>
</tr>
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<tbody>
<tr>
<td>- represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Show ( \frac{1}{5} ) in as many ways as you can?</th>
</tr>
</thead>
</table>

**Materials to be provided:**
- Drawing paper w pencil (area)
- Pattern blocks (area)
- Connecting cubes (volume)
- Counters (set)
- Geoboard (area)
- Coloured tiles (area)
- Hundreds chart (area)
- Cuisenaire rods (linear)
Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Show $\frac{1}{5}$ in as many ways as you can?

Misconceptions Trends
Unit E
Draws an area model of a fraction, where the size of each piece is unequal
Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends
Unit E
Confuses fractions and whole numbers by representing the denominator as a whole number.
Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends
Unit E
Confuses fractions and whole numbers by representing the denominator as a whole number.
Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends
Unit E
Creates a fraction where the numerator and denominator are representing two separate whole numbers.

Show ⅓ in as many ways as you can?
Unit Fractions

Unit E

Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends

Unit E

Creates a fraction where the numerator and denominator are representing two separate whole numbers.
Unit Fractions Misconception Trends

Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Represent $\frac{1}{3}$ in as many ways as you can.

**Misconception Trends Unit E**

- Confuses fractions and whole numbers by representing the denominator as a whole number
  “This is a third”

- Creates a fraction where the numerator and denominator are representing two separate whole numbers
  “This is $\frac{1}{3}$”

- Draws an area model of a fraction, where the size of each piece is unequal
  “This is a third”
# Interview Questions: Unit Fractions

<table>
<thead>
<tr>
<th>Unit Fractions</th>
<th>Grade 4</th>
<th>How could you use this number line to count up by thirds?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit B</td>
<td></td>
<td>Mark the fractions as you count on the line.</td>
</tr>
<tr>
<td>Equally partition area, linear and set models</td>
<td>represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered.</td>
<td>(Have 0 to 1 marked on a number line.)</td>
</tr>
<tr>
<td></td>
<td>Grade 5</td>
<td>How could you use this number line to count up by thirds?</td>
</tr>
<tr>
<td>Unit Fraction</td>
<td></td>
<td>Mark the fractions as you count on the line.</td>
</tr>
<tr>
<td>Unit B</td>
<td></td>
<td>(Have numbers 4 to 6 marked on a number line)</td>
</tr>
<tr>
<td>Equally partition area, linear and set models</td>
<td>represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools</td>
<td></td>
</tr>
</tbody>
</table>
Unit Fractions Supporting Photos

Unit Fractions

Unit B
Equally partition area, linear and set models

Prompt: “Count up by \( \frac{1}{3} \) using this number line”

Misconceptions Trends
Comp B

Does not correctly partition the linear space relative to the fraction being shown.

Does not relate fractional amount to a benchmark whole number.
Unit Fractions Supporting Photos

Unit Fractions
Unit B
Equally partition area, linear and set models

Prompt: “Count up by ¼ using this number line”

Misconceptions
Trends Comp B

Misunderstands “counting up” as involving changing the denominator rather than the numerator.
Unit Fractions Supporting Photos

Unit Fractions

Unit B
Equally partition area, linear and set models

Prompt: “Count up by $\frac{1}{8}$ using this number line”

Misconceptions
Trends Comp B

Confuses the unit fraction with a whole number when counting.
Unit Fractions Misconception Trends

Prompt: “Count up by \(\frac{1}{8}\) using this number line”

**Misconceptions Trends Unit B**

- Confuses the unit fraction with a whole number when counting.
  “1, 2, 3, 4, 5, etc.”

- Misunderstands “counting up” as involving changing the denominator rather than the numerator.
  “1/1, ½ , ⅓ , ¼ , ⅕ , ⅙ , etc.”

- Does not correctly partition the linear space relative to the fraction being shown.

- Does not relate fractional amount to a benchmark whole number.
  “⅓, ⅔, 3/3, 4/3, 5/3, 1”
**Interview Questions: Comparing Fractions**

<table>
<thead>
<tr>
<th>Comp B</th>
<th>Grade 4 - compare fractions to the benchmarks of 0, ½ and 1</th>
<th>Is this closer to 0, ½, or 1?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare familiar fraction quantities with and without benchmark referents</td>
<td>¼</td>
<td>⅜</td>
</tr>
</tbody>
</table>

Explain your thinking.

*(Give number line with benchmarks listed. Give fractions written on post-its to be placed. Prompt to use manipulatives if needed)*
Misconceptions
Trends Comp B
Does not look at the relationship between the size of the numerator and denominator when comparing fractions. Misidentifies the fraction as being larger than it is, because the “bottom number is big”.

Comparing Fractions
Comp B
Compare familiar fraction quantities with and without benchmark referents

Is this closer to 0, ½, or 1?
Comparing Fractions Supporting Photo/Videos

Is this closer to 0, ½, or 1?

1/4

Misconceptions
Trends Comp B
Does not look at the relationship between the size of the numerator and denominator when comparing fractions. Misidentifies the fraction as being larger than it is, because the “bottom number is big”.

Comparing Fractions
Comp B
Compare familiar fraction quantities with and without benchmark referents
Comparing Fractions Misconception Trends

Is \( \frac{1}{4} \) closer to 0, \( \frac{1}{2} \), or 1?

Misconceptions Trends Comp B
- Does not look at the relationship between the size of the numerator and denominator when comparing fractions. Misidentifies the fraction as being larger than it is, because the denominator or “bottom number” is bigger.

“\( \frac{1}{4} \) is closer to 1, because 4 is a big number”
<table>
<thead>
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<th>Interview Questions: Comparing Fractions</th>
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<tbody>
<tr>
<td><strong>Comparing Fractions</strong></td>
</tr>
<tr>
<td>Comp A</td>
</tr>
<tr>
<td>Generate and recognize equivalent fractions using models and symbols</td>
</tr>
<tr>
<td><strong>Grade 4</strong></td>
</tr>
<tr>
<td>– demonstrate and explain the relationship between equivalent fractions, using concrete materials</td>
</tr>
<tr>
<td><strong>Are 2/6 equivalent to 1/3?</strong></td>
</tr>
<tr>
<td>Use the manipulatives or draw to explain.</td>
</tr>
</tbody>
</table>
Comparing Fractions

**Comp A**
Generate and recognize equivalent fractions using models and symbols

**Misconceptions**
**Trends Comp A**
Mistakes the fraction with the larger numerator & denominator as the larger fraction. Does not pay attention to the relationship between numerator & denominator when estimating.
Comparing Fractions Misconception Trends

Are 2/6 equivalent to ½?

Use the manipulatives or draw to explain.

Misconceptions Trends Comp A
- Mistakes the fraction with the larger numerator & denominator as the larger fraction. Does not pay attention to the relationship between numerator & denominator when estimating.
### Interview Questions: Comparing Fractions

<table>
<thead>
<tr>
<th><strong>Comparing Fractions</strong></th>
<th><strong>Comparing Fractions &amp; DECIMALS</strong></th>
</tr>
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<tbody>
<tr>
<td>Comp E</td>
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</tr>
<tr>
<td>Compare fractions with</td>
<td>Compare fractions with</td>
</tr>
<tr>
<td>unlike numerators and</td>
<td>unlike numerators and</td>
</tr>
<tr>
<td>unlike denominators</td>
<td>denominators using models and</td>
</tr>
<tr>
<td>using models and symbols</td>
<td>symbols</td>
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<table>
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<th>Grade 5</th>
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<tbody>
<tr>
<td>- represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools</td>
<td>- represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., number lines) and using standard fractional notation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Which is greater?</th>
<th>Where would these go on the number line?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \frac{2}{3}$ or $\frac{5}{4}$</td>
<td>90/100, 0.3, $\frac{3}{4}$, 0.75</td>
</tr>
</tbody>
</table>

**Show how you know.**

**Explain.**

(Provide a number line with benchmarks: 0, $\frac{1}{2}$, 1. Have the fractions and decimals on post-its for students to place)
Comparing Fractions

Misconceptions
Trends Comp E

Incorrectly judges that a mixed number is always greater than an improper number because of the whole number in front.
Comparing Fractions Supporting Photo/Videos

Comparing Fractions
Comp E
Compare fractions with unlike numerators and unlike denominators using models and symbols

Misconceptions
Trends Comp E
Creates an incorrect proof by creating two fractions that show different models.
Comparing Fractions Misconception Trends

Misconceptions Trends Comp E

- Incorrectly judges that a mixed number is always greater than an improper number because of the whole number in front.
  “1 ⅓ is bigger than 5/4, because it has 1 whole in front”

- Creates an incorrect proof by creating two fractions that show different models.
  “2/8 is bigger, because the circles take up more space in the picture”
### Interview Questions: Fraction Operations

<table>
<thead>
<tr>
<th>Operation with Fractions: Addition &amp; Subtraction</th>
<th>Grade 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP C</td>
<td>- add and subtract fractions with like denominators using models and symbols</td>
</tr>
<tr>
<td>OP D</td>
<td>- add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols</td>
</tr>
<tr>
<td>Give the answer:</td>
<td>$\frac{1}{4} + \frac{1}{4} =$</td>
</tr>
<tr>
<td>Give the answer:</td>
<td>$\frac{1}{2} + \frac{1}{4} =$</td>
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</tbody>
</table>
Operation with Fractions: Addition & Subtraction

OP C
Add and subtract fractions with like denominators using models and symbols

OP D
Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols
Fraction Operations Supporting Photo/Videos

Operation with Fractions: Addition & Subtraction

OP D
Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols

Misconceptions
Trends OP D

Adds numerator and denominator (without creating common denominator)
Operation with Fractions: Addition & Subtraction

OP C
Add and subtract fractions with like denominators using models and symbols

OP D
Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols

Misconceptions Trends OP C
- Adds numerator and denominator (without recognizing like denominators)
  
  e.g. \( \frac{1}{4} + \frac{1}{4} = \frac{2}{8} \)

Misconceptions Trends OP D
- Adds numerator and denominator (without recognizing unlike denominators & creating common denominator)
  
  e.g. \( \frac{1}{2} + \frac{1}{4} = \frac{2}{6} \)
### Operation with Fractions: Addition & Subtraction

<table>
<thead>
<tr>
<th>Grade 7</th>
<th>Estimate the answer: $\frac{12}{13} + \frac{7}{8}$</th>
</tr>
</thead>
</table>
| - add and subtract fractions with simple like and unlike denominators, using a variety of tools and algorithms | A. 1  
B. 2  
C. 19  
D. 21 |
Operation with Fractions: Addition & Subtraction

OP E
Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols

Misconceptions Trends OP E
•
Operation with Fractions: Addition & Subtraction

OP E
Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols.
Operation with Fractions:
Addition & Subtraction
OP E
Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols

Misconceptions Trends OP E
- Adds numerators without estimating or creating common denominator
e.g. \( \frac{12}{13} + \frac{7}{8} = 19 \) “because 12 + 7 = 19”
- Adds denominators without estimating or creating common denominator
e.g. \( \frac{12}{13} + \frac{7}{8} = 21 \) “because 13 + 8 = 21”

Estimate the answer: \( \frac{12}{13} + \frac{7}{8} \)
A. 1  B. 2  C. \( \frac{13}{19} \)  D. \( \frac{8}{21} \)