



Tips for Teachers

*Critical thinking
fosters strong
reasoning in math.*

Deepening mathematical reasoning

The key to improved mathematical reasoning is nurturing a student's ability to make thoughtful and defensible judgments. By tweaking our goals and questions, teachers can help students develop powerful tools to deepen their mathematical reasoning. Teachers can enhance student reasoning in mathematics by:

- including an evaluative (student decision-making) component in the task or question
- being very specific in the questions asked
- articulating the criteria students should consider
- requiring precisely explained evidence, reasoning or proof to support a conclusion

The following examples illustrate how common goals and tasks in mathematics can be enriched using these strategies.

Common goals and tasks

Comprehend a concept:

- Describe the Cartesian coordinate plane.

Identify similarities and differences:

- What is similar about line graphs and scatterplots?
- What is different about them?

Identify the implication of changed variables:

- How did the inclusion of negative numbers impact the product of the multiplication question?

Enriched goals and tasks

Understand a concept with sufficient depth and accuracy to evaluate its most important or challenging dimensions:

- *What are the five most important things to know about the Cartesian coordinate plane?*
- *Predict those aspects of the Cartesian coordinate plane that might be confusing for someone first encountering this concept, and write a short list flagging potential misunderstandings.*

Deepen understanding of a concept through detailed and nuanced consideration of similarities and differences:

- *Are scatterplots and line graphs similar enough that you could use either form to communicate the same information? Which is the more accurate way to present the information?*

Carefully consider, assess and predict the implications of changed variables:

- *How could you use negative integers to have the biggest impact on a final multiplication solution?*
- *How could you minimize the impact of negative integers?*

Communicate understanding of mathematical processes:

- Why do you think subtracting a negative integer results in a positive movement along the number line?

Communicate an in-depth understanding of mathematical processes through evidence-based reasoning:

- Provide an evidence-based argument, which can include visual representations, explaining why the subtraction of a negative number results in a positive movement along the number line.

Exchange ideas with peers to enhance mathematical understanding:

- How many ways could you accurately represent the Pythagorean theorem?

Evaluate ideas according to established criteria to enhance mathematical understanding:

- Establish criteria for an effective mathematical representation. Based on these criteria, which visual model from your brainstormed series would you select to represent the theorem effectively in a mathematics poster for all students? Provide reasons why this visual model is the most effective for this purpose.

Identify the properties and measurement algorithms of various geometric shapes:

- Describe the mathematical properties of a cylinder.
- Calculate the surface area and volume of the cylinder.

Develop conceptual understanding of geometric algorithms in a rich context:

- What would you need to know in order to calculate how long it would take to fill this cylinder with water?

Choose among possible solutions:

- Which solutions look good?

Use criteria to evaluate and choose among solutions:

- What would distinguish a good solution from a mediocre solution? Establish criteria for an effective solution. Which of these two solutions better meets the criteria?

Improve mathematical communication:

- Explain your solution to a partner.

Use criteria to reflect upon and improve mathematical communication:

- What might you do to explain your solution clearly to someone?

Determine the accuracy of solutions:

- Is your answer correct?

Evaluate the accuracy of solutions using precise mathematical language and focused, criteria-based self-assessment:

- How do you know your answer is reasonable? Using mathematical terms, inferences and evidence, justify your answer.