Measurement: Part 2

Building the Measurement Instructional Continuum at Brock

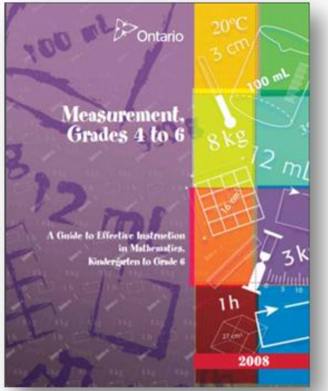
Extra! Extra! Chocolongo Chocolate Bar Company Criticized!

The Chocolongo Chocolate Bar has been a favourite for years, but since the public is becoming increasingly concerned about the impact that food packaging has on the environment, sales have plummeted!

This is very distressing news for the company. The president of th

company has called a crisis management meeting of her strategy team.

The Chocolongo Company vows to reduce packaging on its chocolate bar, while still providing the same great amount of chocolate.



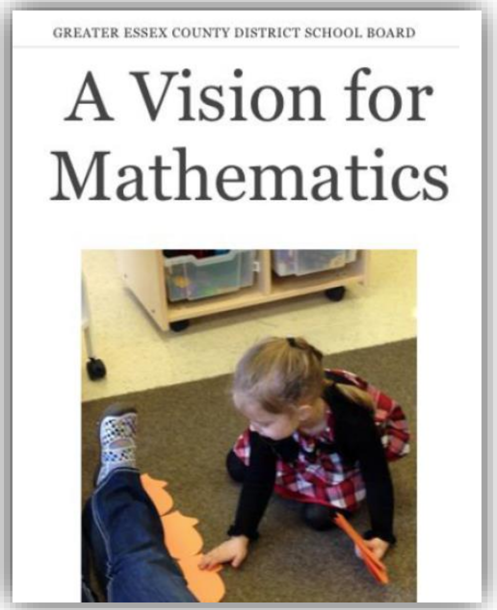
You are a member of the crisis management team and you must come up with a new format for the Chocolongo Bar.
You have 36 cubes, which represent the volume of the Chocolongo Bar.

Your task is to work with the 36 cubes to find all other possible formats for the new and improved bar.
You must provide proof that the selected format will result in the least amount of packaging.

For shipping and storage purposes, the final product must be in the form of a rectangular prism.

Good luck...the future of the Chocolongo Bar Company is in your hands!

The Work: Ambitious and Necessary

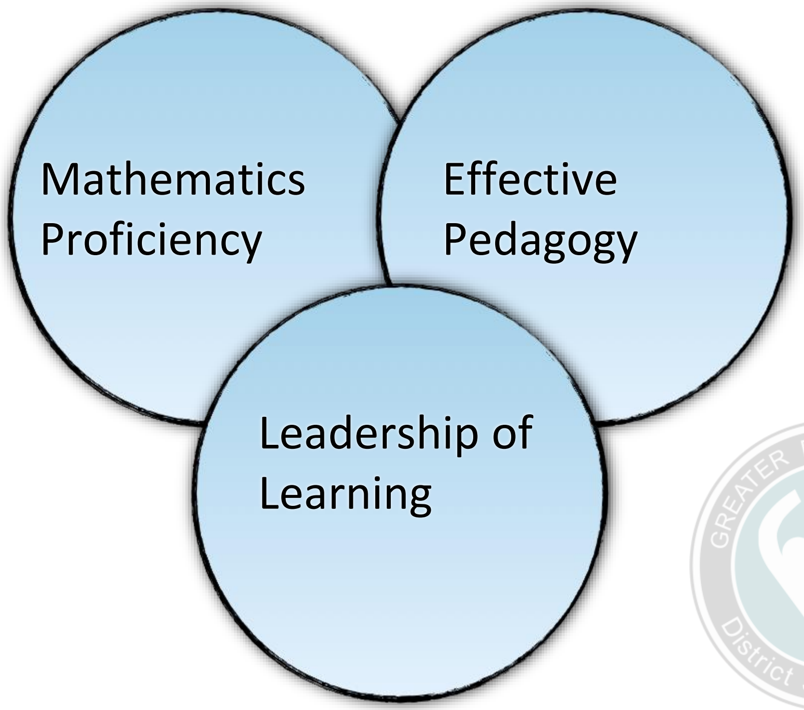


Enact the Vision

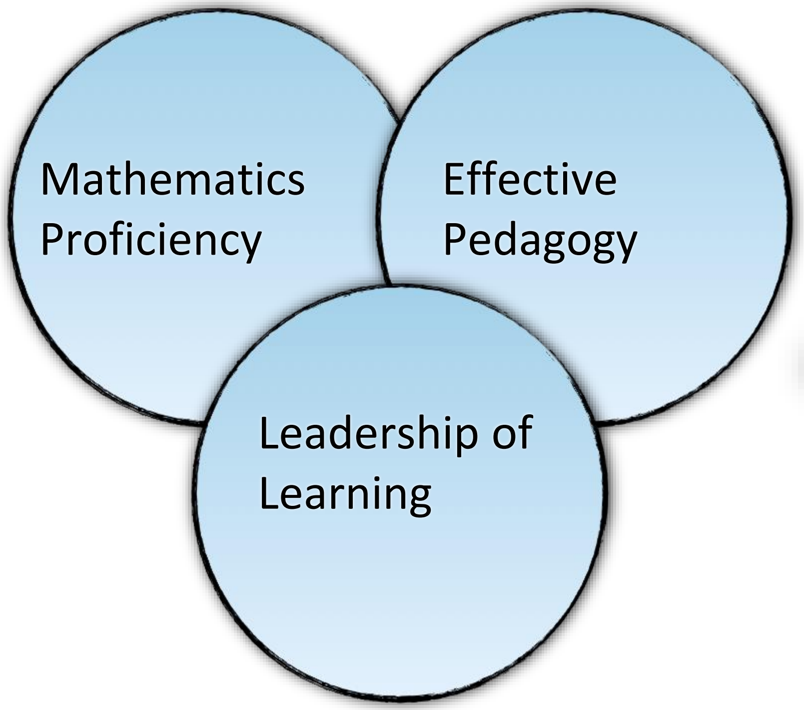
"The GECDSB provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics."

2016-17 GECDSB Math Strategy Ta

- Mathematical Proficiency
- Meaningful Manipulative Use
- Math Learning Continuums
- Curriculum Connections
- Pedagogical System
- Concreteness Fading
- Substance vs Structure
- Spatial Reasoning
- Assessment for Learning



2017-18 GECDSB Math Strategy



CONCEPTUAL UNDERSTANDING

Ability to understand mathematical concepts, operations, and relationships

PROCEDURAL FLUENCY

Understanding and using a variety of mathematical procedures

ADAPTIVE REASONING

Capacity for logical thought, reflection, explanation, and justification

STRATEGIC COMPETENCE

Ability to formulate, represent & solve mathematical problems using an effective strategy

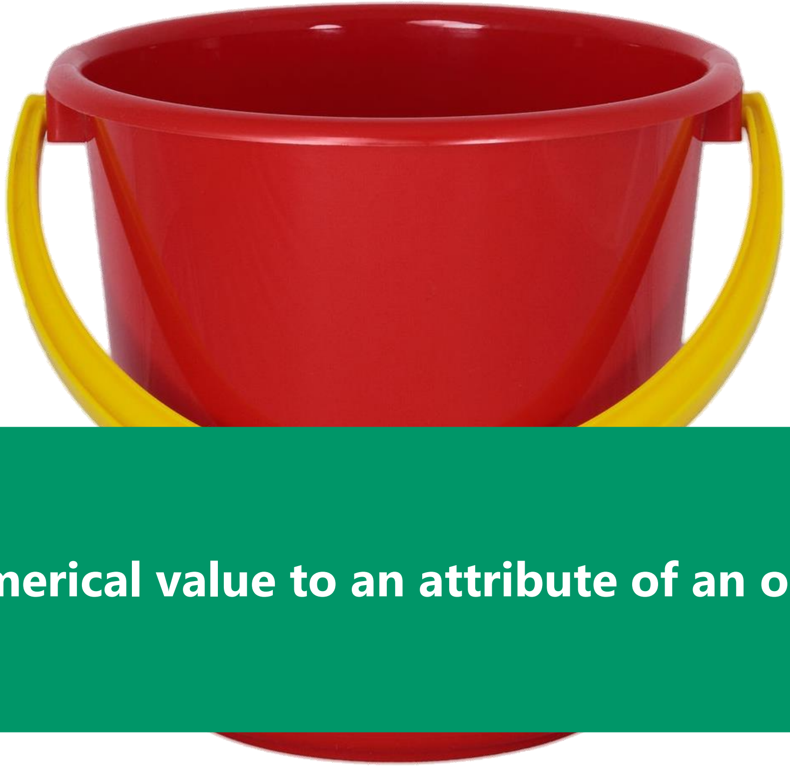
PRODUCTIVE DISPOSITION

Inclination to see mathematics as useful and valuable.

How can you measure a bucket?

Consider:

- attributes
- possible units



Measurement = assigning a numerical value to an attribute of an object

Our Plans for Today...

Process of Measurement
Look at Direct and Indirect Measurement: THE UNIT
What is a unit?
Understanding the Unit
Isaac’s work- Big Ideas & Misconceptions

Task Breakout- Measuring the Leaf
Units are qunatities not shapes...
Moving to Standard units

Process of Measu

1. Decide attribute to be measured
2. Select a unit that has the same attribute
3. Compare the units - by filling, covering, matching or using known measures. Recognize the number of units needed = the measurement

Implications for Measurement Instruction

1. Students must understand the attribute to be measured = direct comparisons

2. Students must understand how filling, covering, matching etc. produces a number called a measure = use physical models (non-standard units)

3.Students use common measuring tools with understanding and flexibility = make measuring tools, then connect to standard tools

Continuum of Measurement Understanding

Impacts measurement *instruction*...we need to sequence experiences

1) Direct COMPARISONS: Consider the two shapes at your table: Using only the shapes themselves, which shape is bigger?

2)Indirect comparisons: Now consider the same two shapes: Using only the shapes themselves and the colour tiles provided, be more precise. Which shape is bigger?

3) Direct Measurements: Now consider the same two shapes: Using only the shapes themselves and a ruler, be even more precise. Which shape is bigger?

4) Indirect Measurement: Now consider the same two shapes: Using only the shapes themselves, known measurements, be even more precise. What is the area of each shape?

What is a unit?

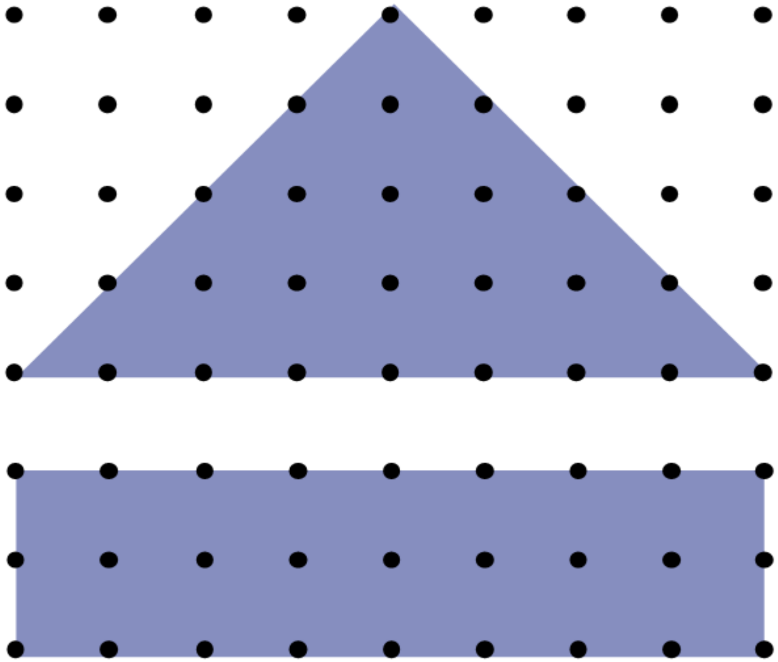
- As soon as we move from comparing to measuring, as soon as we start quantifying a comparison, we need to use a unit. The whole idea of “units” is huge in mathematics, one of those big overarching themes that crosses a bunch of strands.

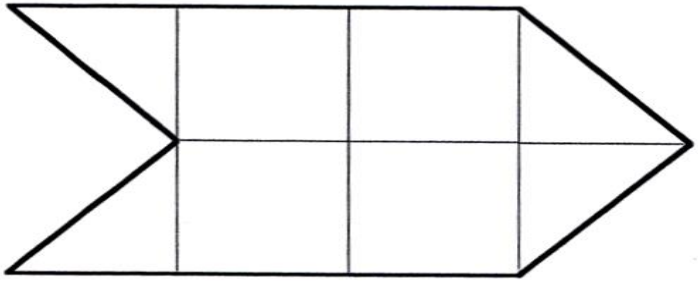
- We talk about unit fractions (one-fifth, one-tenth, one-eighth); we see it in place value when we look to the ones column -- the unit -- and realize that everything, both whole numbers and decimals, are centered around that unit.

- We see it in proportional reasoning, when we find the unit rate to compare ratios.

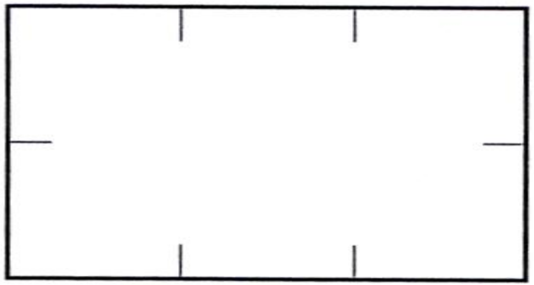
- We see it at the heart of measurement. Linear units, like a centimeter or inch; units of area, like square centimetres or square kilometers; units of volume like a cup or a litre; units of time, like minutes or hours.

Understanding the Unit:





A

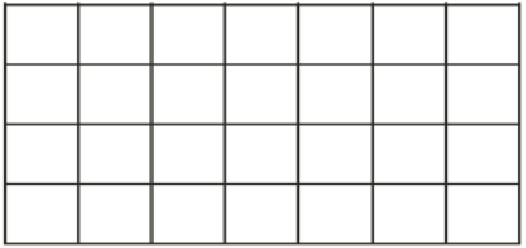


B

Isaac's Area Work

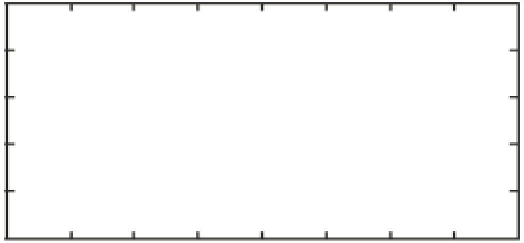
What Is the Area?
A number of Grade 7 students were interviewed individually and asked to find the area of some shapes. After completing each one, they were asked to explain how they worked it out. Here are Isaac's responses to the tasks.

A



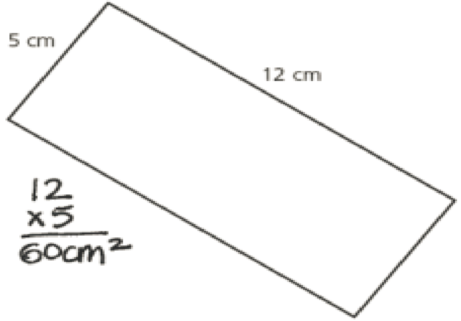
I: 28 squares.
T: How did you work that out?
I: Four down the sides and 7 across the top, and I think that equals 28.

B



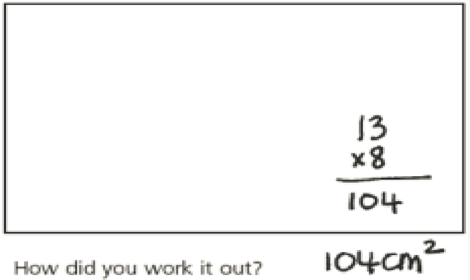
I: 40 squares.
T: How did you work that out?
I: If we joined the lines up, there would be squares. So there's 5 down the sides and there's 8 squares across the top. So 8 times 5 is 40.

C



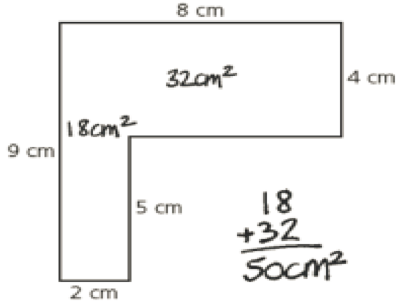
I: I worked out 5 times 12 is 60 centimetres. I mean centimetres squared.
T: Can you tell me why you multiplied the numbers?
I: If I worked it out, it would take a bit longer to count them out. If I know the measurement on each side and multiply, it's quick.

D



T: How did you work it out?
I: I measured all the centimetres. There's 13 across the bottom and 8 across the side, so 8 times 13 is 104 centimetres.
T: What did you want to call the measure?
I: Centimetres squared. *(rewrote the answer)*

E



What would you say Isaac knows about area?

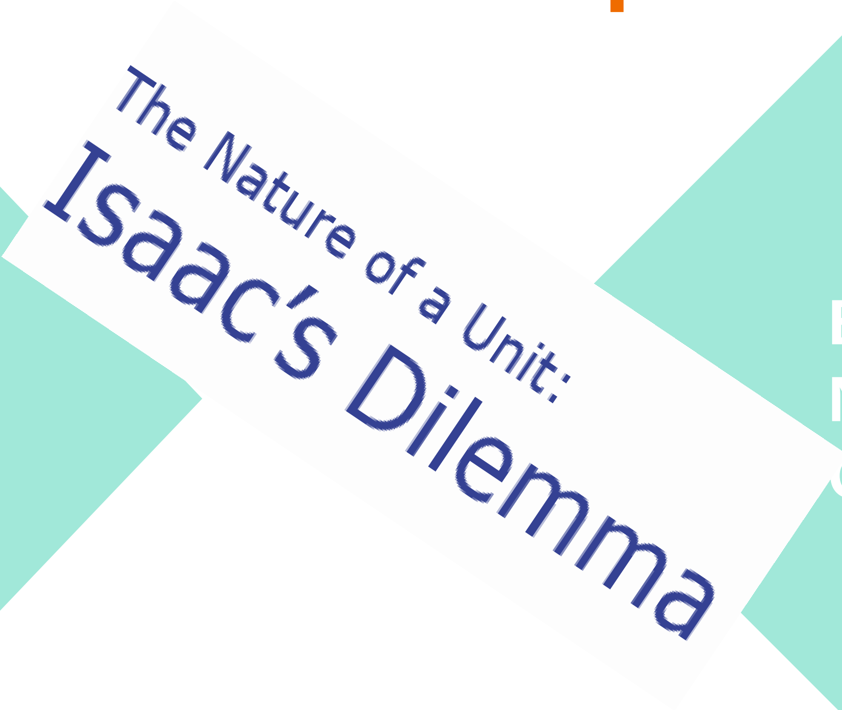
Measure the area of this leaf:

Show or explain how you did it.

Big Idea #3 Units are quantities not shapes …

BIG IDEA 1

Constant Units



BIG IDEA 2
No Gaps or Overlaps

Units are quantities not shapes

A lot of times because kids are only confronted with squares when they're working with area, and because they're called square units, we think that a square unit is a square. But really it is a quantity -- an amount of space that can be re-assembled any way we wish. If Isaac had realized that these shapes could be cut and reshaped – as long as he conserved the area of that unit – he would have been fine. But he realize didn't that units are like liquid – they're fluid – they can morph into any shape as long as the quantity doesn't change.

It's obvious when we're talking about capacity. A litre can come in all sorts of different shapes. Right? But with area -- I think it's because we tend to give kids fixed materials like square tiles or a grid that is made up of squares -- we don't bump up against the idea that a square unit can be all sorts of different shapes.

Quantities not shapes….

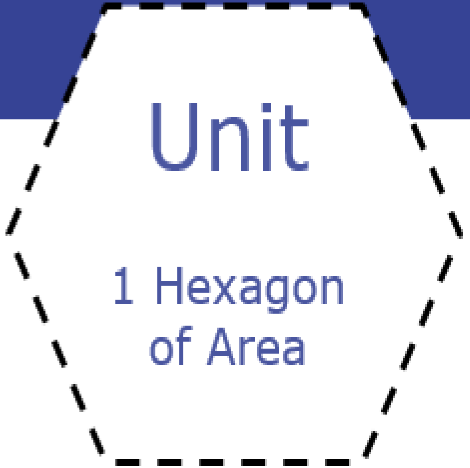
Unit

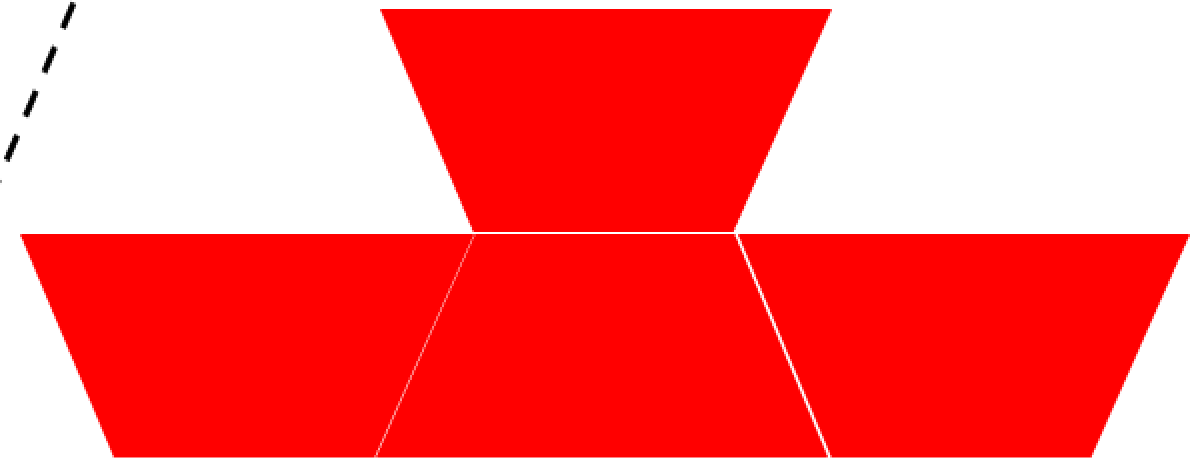
1 Hexagon of Area

The Object or Instrument

we use to measure 1 Hexagon of area

What is the area of this boat?





2 Hexagons

Five Big Ideas about the Unit and Direct Measurement	
1	To measure something is to say how much of a particular attribute it has.
2	The unit we choose determines our level of accuracy. The smaller the unit, the more accurate our measure can be.
3	A unit is a quantity, not a shape.
4	We choose an *instrument* to concretely represent a unit. We can use different instruments to represent that unit so long as the *quantity* is the same.

And lastly…..

5	The *accuracy* of our measurement is determined by: 1. how well our instrument *matches* the the unit; 2. how *consistently* we *repeat* our instrument; 3. how *completely* we *match* our instrument to what we're measuring.

Standard Units….History of Measurement:

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Moving to the Standard Unit:

The Relationship between Place Value and the Metric System

	10^{12}	10^{9}	10^{6}		10^{3}	10^{2}	10^{1}	Root unit	10^{-1}	10^{-2}	10^{-3}		10^{-6}	10^{-9}	10^{-12}
Place values	trillions	billions	millions		thousands	hundreds	tens	unit (ones)	tenths	hundreths	thousandths		Millionths	billionths	trillionths
Metric prefix	tera (T)	giga (G)	mega (M)		kilo (k)	hecto (h)	deca (da)		deci (d)	centi (c)	milli (m)		micro (μ)	nano (n)	pico (p)
Length	terametre (Tm)	gigametre (Gm)	megametre (Mm)		kilometre (km)	hectometre (hm)	decametre (dam)	metre (m)	decimetre (dm)	centimetre (cm)	millimetre (mm)		micrometre (μm)	nanometre (nm)	picometre (pm)
Capacity	teralitre (TL)	gigalitre (GL)	megalitre (ML)		kilolitre (kL)	hectolitre (hL)	decalitre (daL)	litre (L)	decilitre (dL)	centilitre (cL)	millilitre (mL)		microlitre (μL)	nanolitre (nL)	picolitre (pL)
Mass	teragram (Tg)	gigagram (Gg)	megagram (Mg)		kilogram (kg)	hectogram (hg)	decagram (dag)	gram (g)	decigram (dg)	centigram (cg)	milligram (mg)		microgram (μg)	nanogram (ng)	picogram (pg)

These prefixes are also used with other units such as power (watts), pressure (pascal), frequency (hertz) and energy (joule).

About One Unit

Give students physical model of a unit and have them search for objects that have the same measure as that 'one unit'. Then extend to bigger than and smaller than...

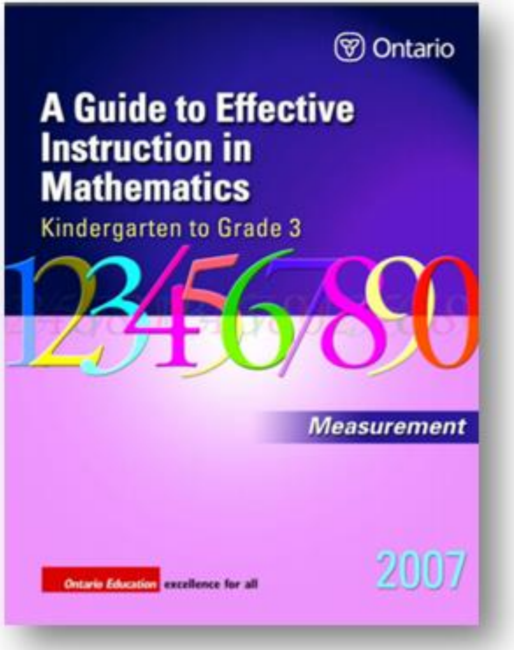
Personal benchmarks

Guess the Unit

Find examples of of measurements (newspapers, signs etc.). Present the context and measure, but not the units...guess the unit.

Making and Using Rulers

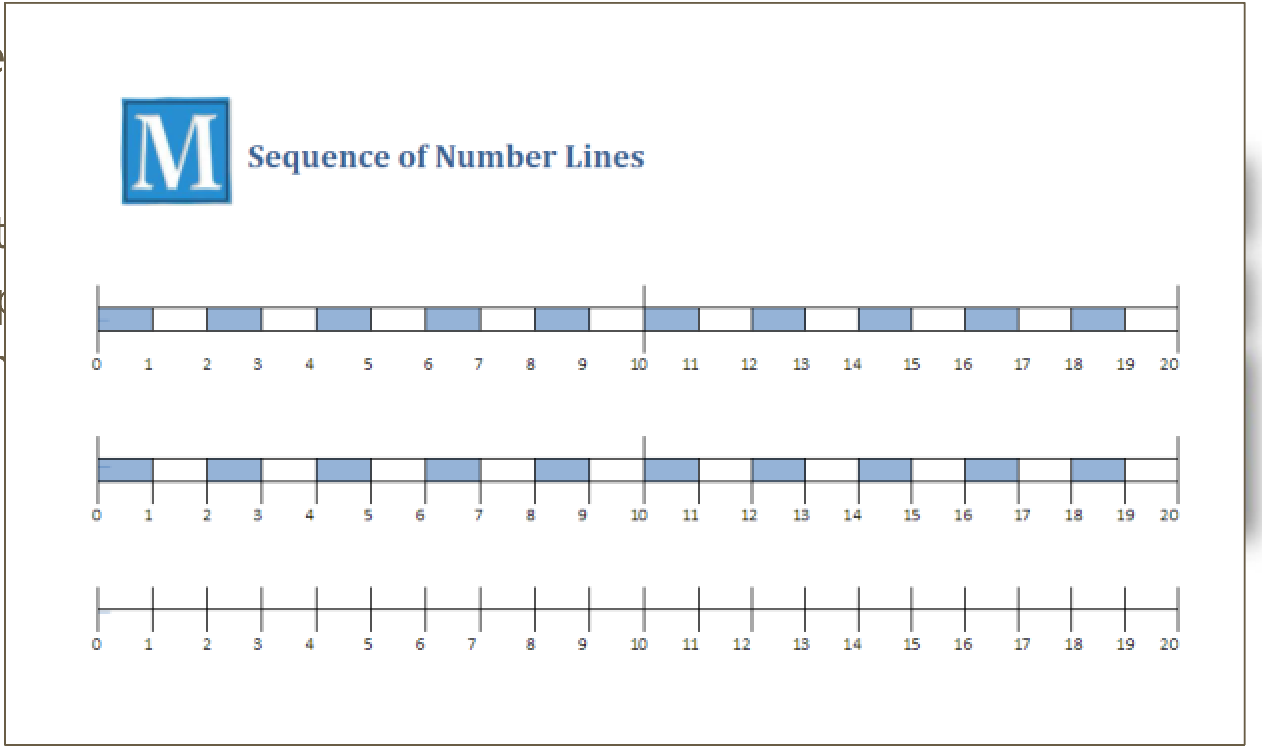
If the Shoe Fits



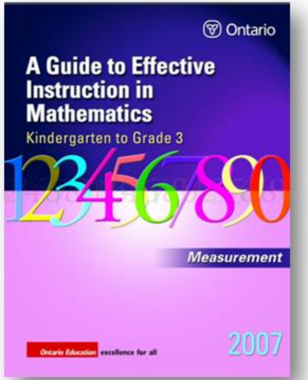
Connecting Number Lines & Rulers

Sequence of number line
counting of the unit.

1. Glue units on card st
2. Add numbers to help
3. Standard rulers: nur



Understanding the Passage of Ti

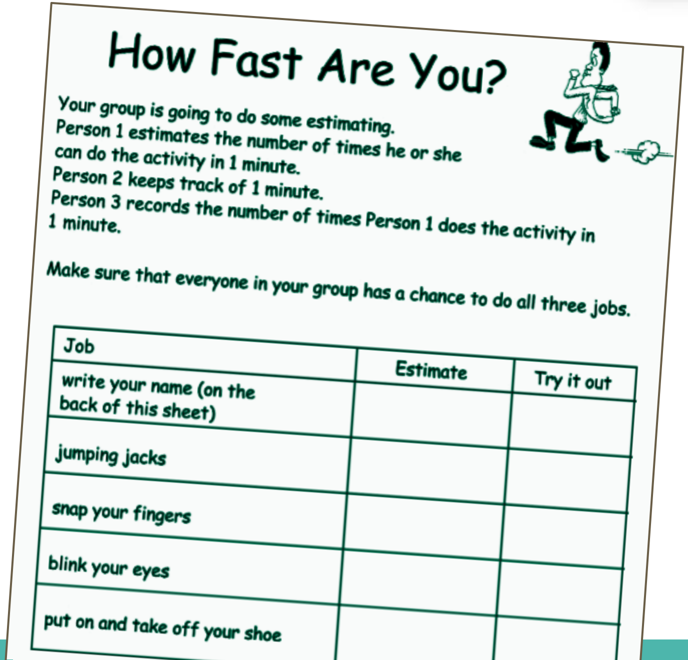


The passage of time is different than reading a clock.

This understanding develops through:

- personal benchmarks
- comparing events

See the *Guides to Effective Instruction…*



How Fast Are You?

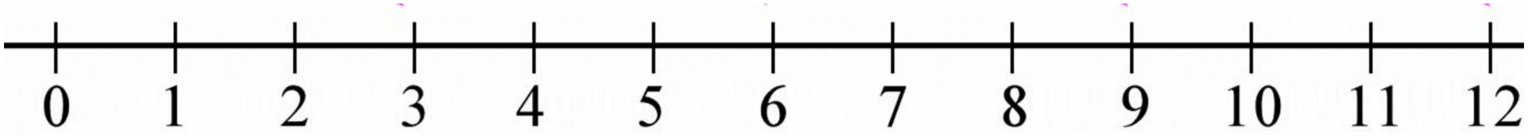
Your group is going to do some estimating.
Person 1 estimates the number of times he or she can do the activity in 1 minute.
Person 2 keeps track of 1 minute.
Person 3 records the number of times Person 1 does the activity in 1 minute.

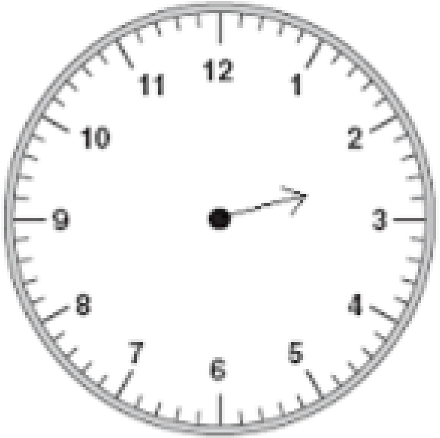
Make sure that everyone in your group has a chance to do all three jobs.

Job	Estimate	Try it out
write your name (on the back of this sheet)		
jumping jacks		
snap your fingers		
blink your eyes		
put on and take off your shoe		

Reading Analogue Clocks

Re-thinking our time "unit"

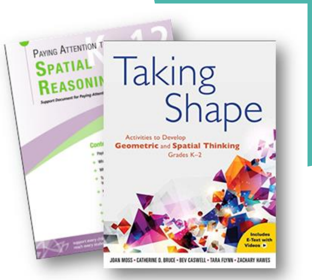




Two-Piece Shapes

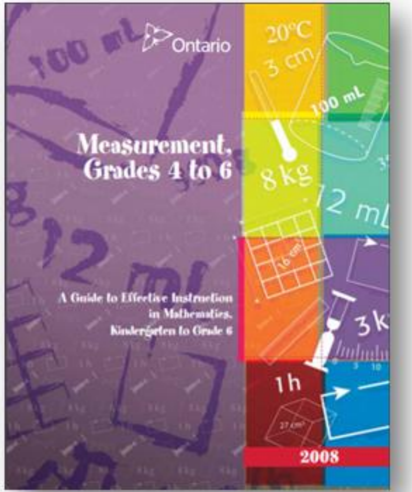
Fill and Compare

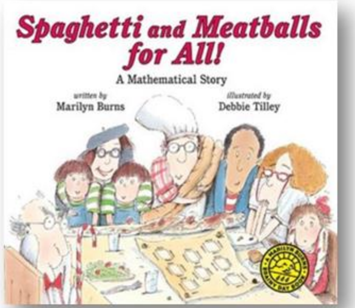
Hit the Target

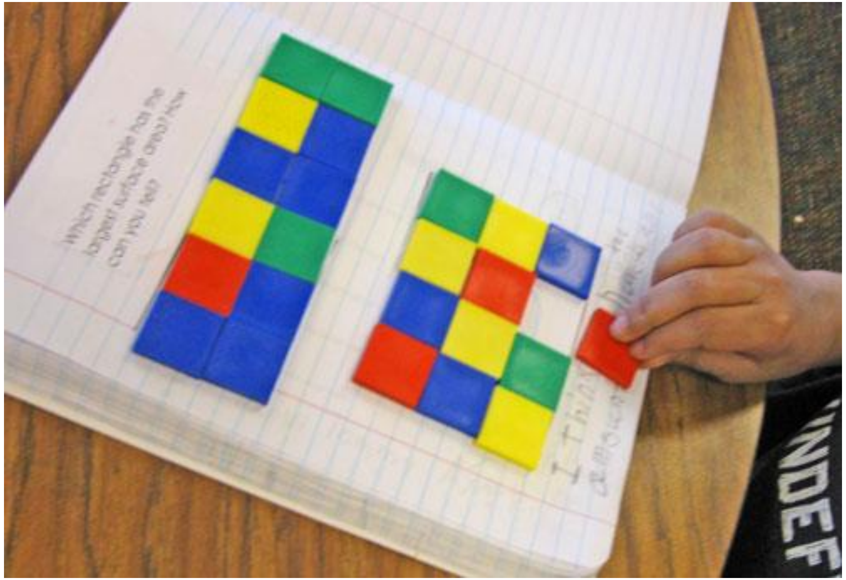


Activating physical models

Fixed Area/Fixed Perimeter Lessons







Developing Formulas

When students develop formulas, they gain conceptual understanding of the ideas and relationships involved, and they engage in ‘doing mathematics’.

Formulas = generalizations

Generalizing = algebraic reasoning

Exploring measurement relationships = forming conjectures

Proving conjectures = algebraic reasoning

Let's explore rectangular prisms!

Anticipate, Select, Sequence & Connect

- **Concrete model** – count squares
- **Pictorial model** – draw isometric drawings, and/or nets
  - Find area of different sides, add
  - Use strategies like x2 , recognizing that opposite faces are the same
- **Numerical model** – records in table

- Recognizes 3 dimensions multiply to equal 36
- Notices pattern – closer to cube, smaller surface area = generalize

Estimation & Approximation

“Measurement estimation is the process of using mental and visual information to measure.”

Van de Walle, p. 276

Estimation is about reasoning and reasonableness …it is very mathematical!

- estimation helps children focus on the attribute being measured
- estimation provides intrinsic motivation for measurement
- estimation develops familiarity with standard units
- estimation lays the foundation for multiplicative thinking

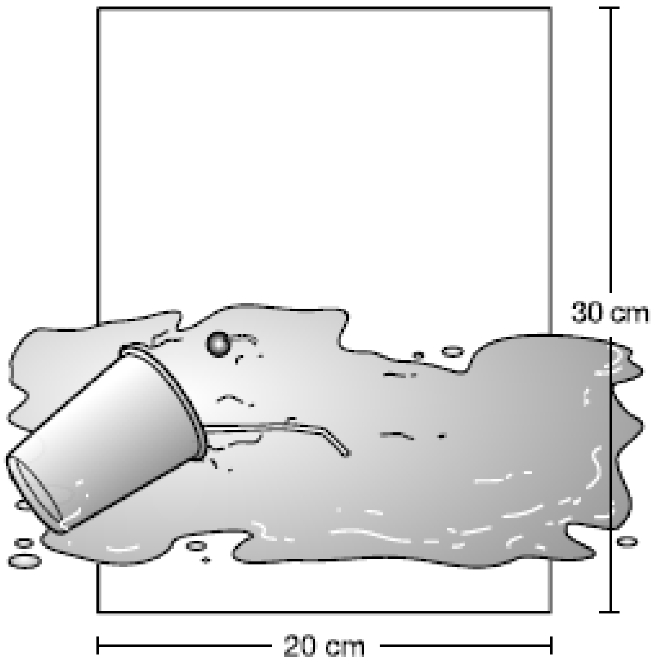
…benchmarks are an important part of

Estimation & Approxim

- You try:

Oh drat ...we have no formula for an irregular milkshake spill! What shall we do?

Samantha spills a milkshake on a rectangular piece of paper as shown below.



Which of the following best approximates the area of the entire spill?

Activating and Developing Estimation

1. Develop and use personal benchmarks
2. Use chunking or subdivisions
3. Iterate a unit mentally or physically

Estimation tasks are a good way to assess students' understanding of both measurement and units.

D

E

Make estimation an ongoing activity.

Problem Sort

Sort the collection of problems:

KNOWLEDGE THINKING

APPLICATION

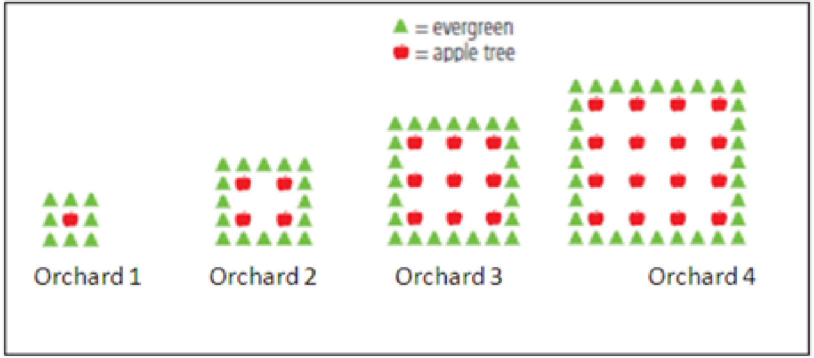
Where does communication fit?

Whole School Task – Algebraic Thinking

Name:

My neighbor plants his apple trees in a square pattern in each orchard. To protect the trees from the wind, he plants evergreens all around the orchard.

How many of each type of tree will there be in Orchard 6?



Continuum of Thinking: Whole School Task – How many of each tree in orchard #6?

Build/Draw/Count	Recursive Pattern	Functional Thinking	Second Differences	Algebraic Expressions
The student : • builds or draws to continue the pattern • counts to determine the number of trees in orchard #6 • building/drawing and counting will have different levels of accuracy.	The student: • creates a table recording numbers of trees for each term given • recognizes the pattern "down" the table • uses the pattern going down the table to continue the table to determine the number of trees in orchard#6	The student: • creates a table recording number of trees for each term given • recognizes a "pattern rule" that relates one column to the next • uses the pattern rule to determine number of trees in orchard #6	The student: • creates a table to record number of trees for each term given • examines first and second differences • uses first and second differences to determine number of trees in orchard #6 or to develop pattern rule	The student: • uses algebraic expressions to represent number of trees and determine number of trees in orchard #6 • recognizes that different representations can be used