


Progression of Fractions





March 23: Brock PS

Plans for today:

Fractions: What do you notice or wonder??

Carton Counting

Review of Paying Attention to Fractions

Doritos---- Hot or Not (Fraction Constructs)

Baking Brownies– unfriendly Fractions

Rolos--- How many pieces?

Fraction Models

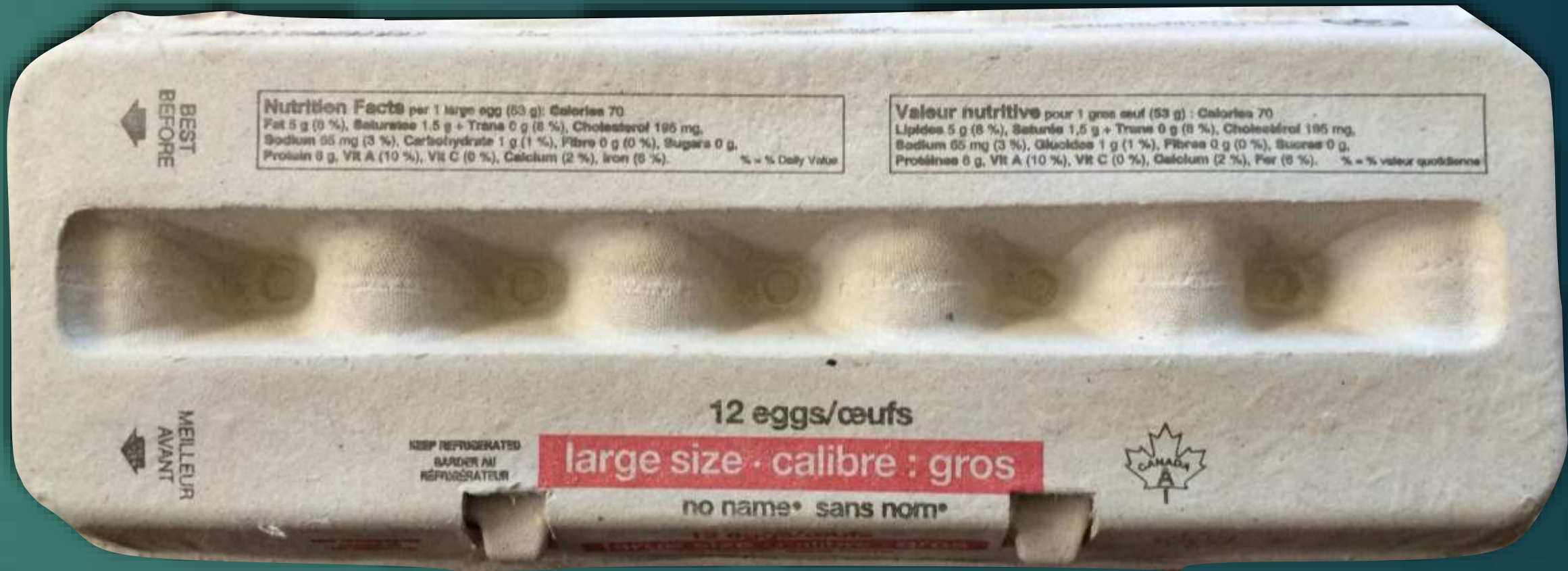
LET'S DO SOME MATH!!!!

Attendance:

- ▶ Anne Adamson (P)
- ▶ Bryan Johnson (VP)
- ▶ Stephanie Douglas
- ▶ Cecilia Joseph
- ▶ Sheri Gaetz
- ▶ Kristi Jasey
- ▶ Danilela Amato
- ▶ Monica Mackenzie
- ▶ Mark Patterson
- ▶ Alana Parsons
- ▶ Kristine Bentley
- ▶ Rebecca Davies
- ▶ Stephanie Molenda
- ▶ Brian Bartlett
- ▶ Andrea Manchen
- ▶ Pauline Brown
- ▶ Jennifer Kathen Groggin
- ▶ Matina Lousisa
- ▶ Leah Beherns
- ▶ Giovanna Giglio



What do you...



What do you...

Notice and Wonder?



How many eggs are there?



How many eggs are there?



Carton Counting



How many cartons are there?



Carton Counting



How many cartons are there?



Carton Counting



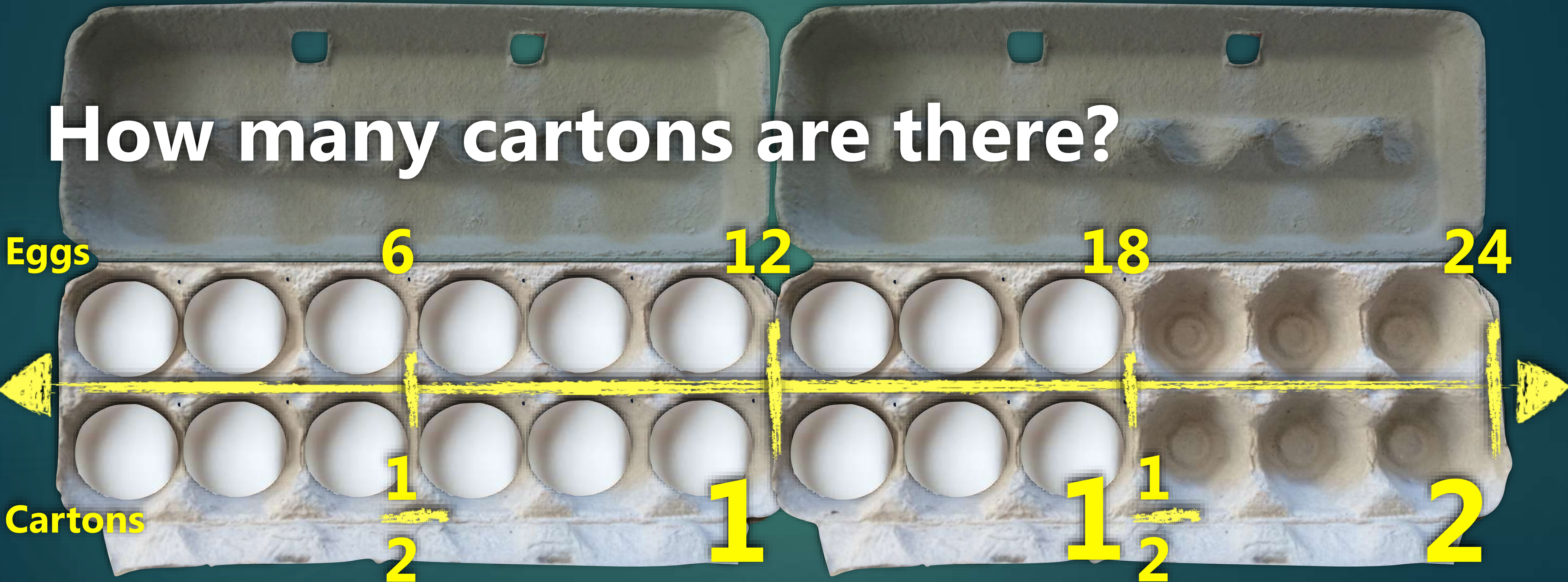
How many cartons are there?



Carton Counting



How many cartons are there?



Eggs

6

12

18

24

Cartons

1

1

1

1

2

2

2

Carton Counting



How many cartons are there?



Carton Counting



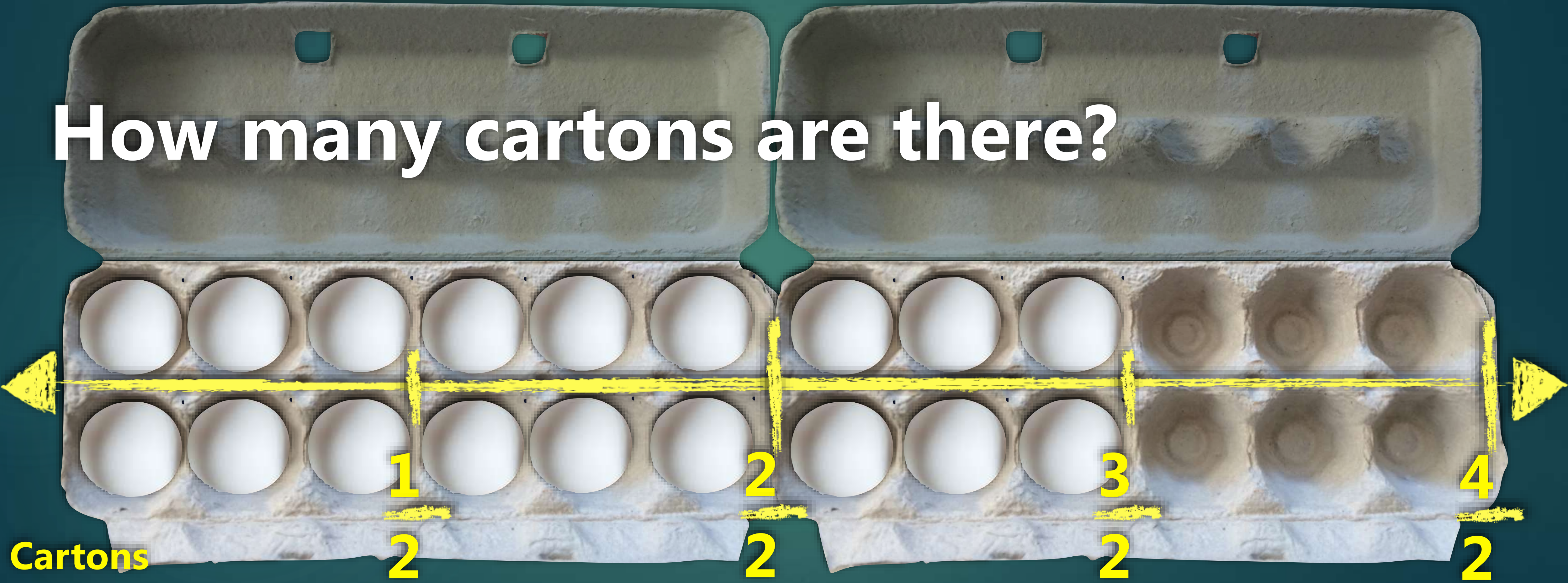
How many cartons are there?



Carton Counting



How many cartons are there?



Cartons

1
2

2
2

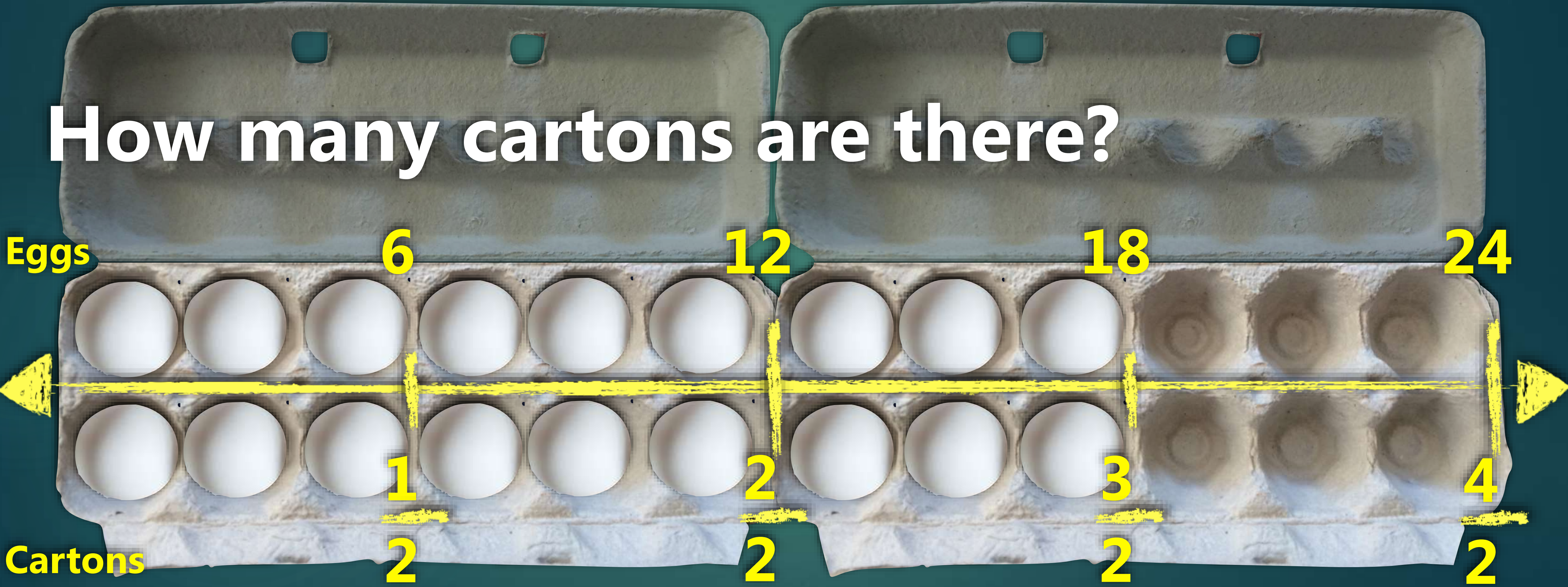
3
2

4
2

Carton Counting



How many cartons are there?



PAYING ATTENTION TO **K-12** **FRACTIONS**

Support Document for Paying Attention to Mathematics Education

Contents

- ❖ Paying Attention to Fractions
- ❖ Why Is Understanding Fractions Important?
- ❖ What Is a Fraction?
- ❖ Exploring Key Concepts
- ❖ How Can We Promote Fractions Thinking?
- ❖ Fractions across Strands and Grades
- ❖ Ministry Resources
- ❖ References

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Paying Attention to Fractions



“The research suggests that explicit and precise changes to learning and teaching practices can have a substantial impact on children’s **understanding of fractions** and **future mathematical success**. Instructional decisions have a significant bearing upon students’ ability to understand the concept of fractions, including the ability to represent fractions appropriately, compare the relative magnitude of two fractions, and complete calculations accurately.”

(Bruce, Chang, Flynn & Yearley, 2013, p. 32)

Why Is Understanding Fractions Important?

“No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios, and proportionality. These ideas all express mathematical relationships: fractions and ratios are ‘relational’ numbers. They are the first place in which students encounter numerals like ‘ $3/4$ ’ that represent relationships between two discrete or continuous quantities, rather than a single discrete (‘three apples’) or continuous quantity (‘4 inches of rope’).”

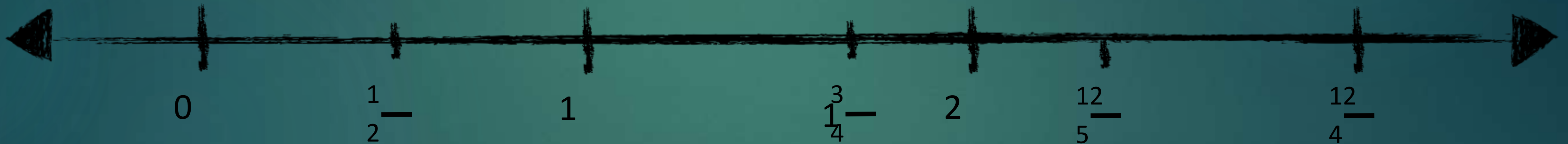
(Litwiller & Bright, 2002, p. 3)

What is a Fraction?



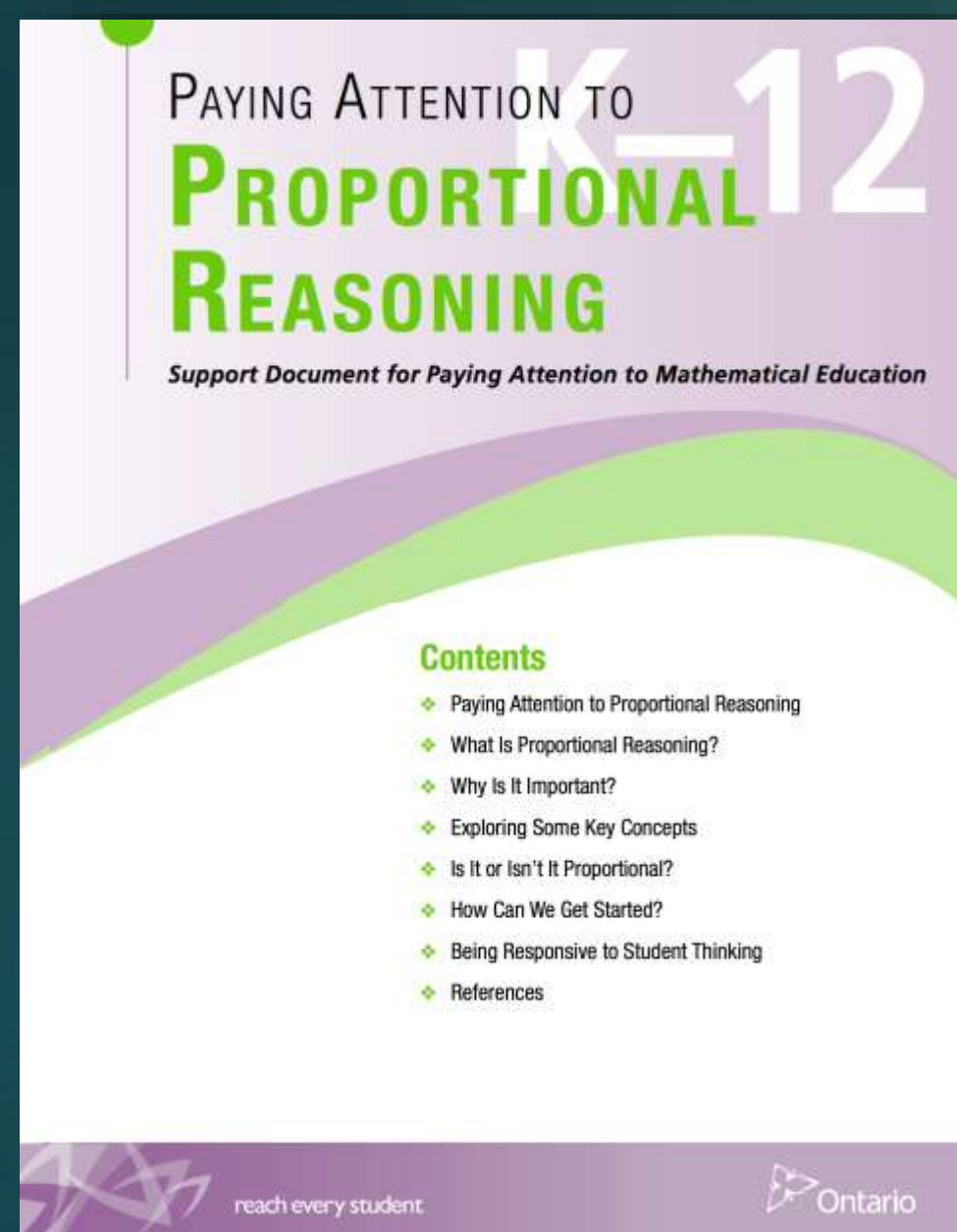
A fraction is a **number**.

While fractional notation is typically used to represent quantities that are **not whole**, it is possible for all quantities to be represented as a fraction.



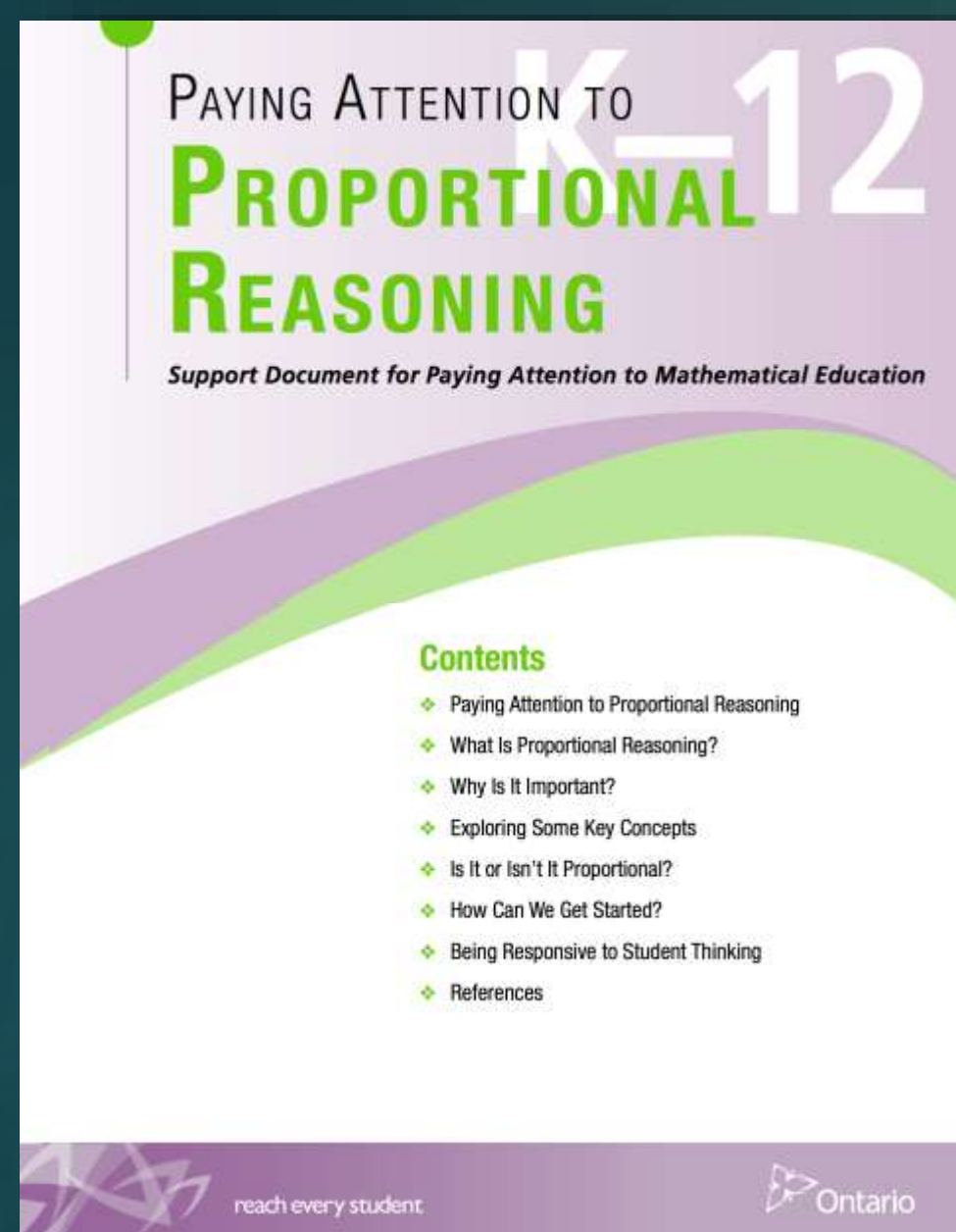
These simple descriptions do not appropriately communicate the complex constructs that lie within this big idea.

Connecting Fractions and Proportional Reasoning



“The essence of proportional reasoning is the consideration of number in relative terms, rather than absolute terms.”

Connecting Fractions and Proportional Reasoning



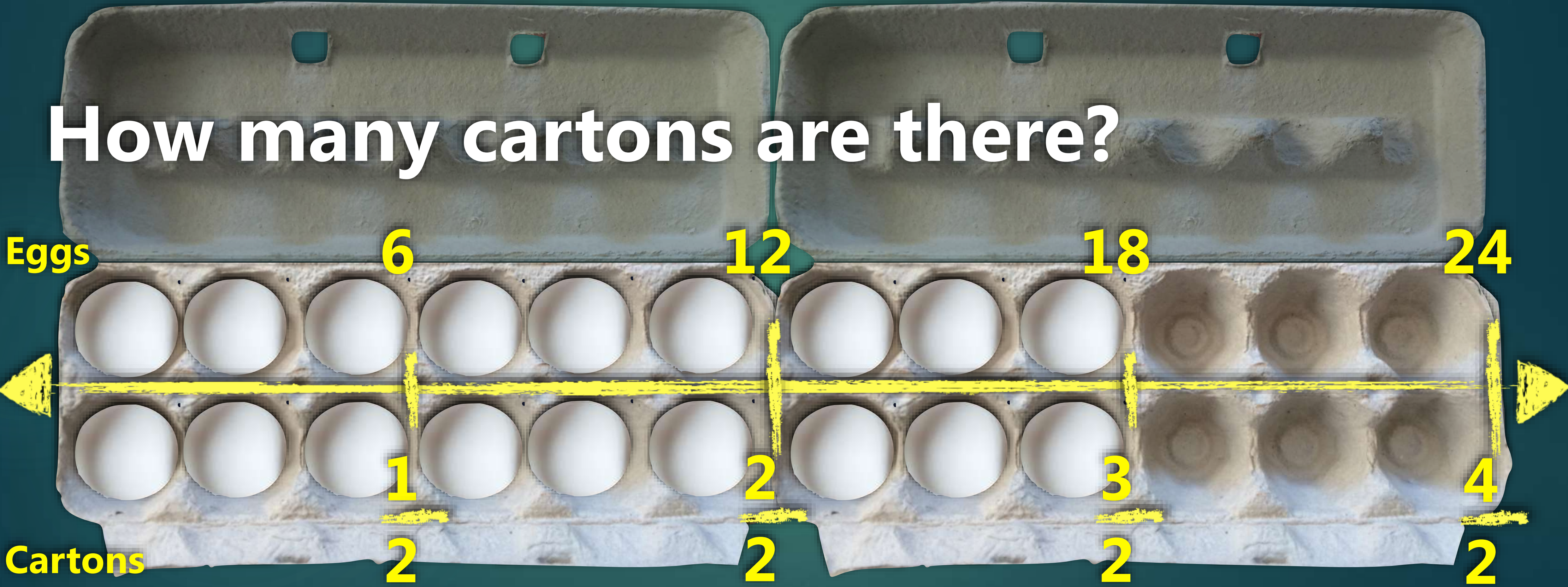
“The essence of proportional reasoning is the consideration of number in relative terms, rather than absolute terms.”



Part-Whole Relationship



How many cartons are there?



Trans Fat
0 gras trans

Doritos

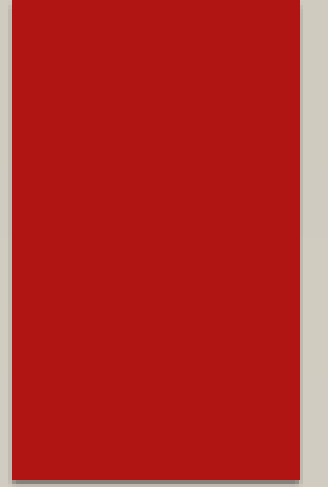
Roulette



ATTENTION
SOME CHIPS VERY HOT
CERTAINES CHIPS SONT VRAIMENT ÉPICES

TORTILLA CHIPS
CHIPS TORTILLA AROMATISÉES

What do you...



Notice?

Wonder?

Doritos

Roulette

FRAÎCHEUR GARANTIE
GUARANTEED FRESH

• UNTIL PRINTED DATE •
• JUSQU'À LA DATE INDICUÉE •

DESA

3032 2954

21:36

ATTENTION
SOME CHIPS VERY HOT
CERTAINES CHIPS SONT VRAIMENT ÉPICÉES

PRODUCT ENLARGE
SHOW TEXTURE
PRODUIT AGRANDI
MONTRER LA TEXTURE

FLAVOURED TORTILLA CHIPS
CHIPS TORTILLA AROMATISÉES

235

Nutrition Facts Valeur nutritive

Per 21 chips (50 g)
pour 21 croustilles (50 g)

Amount Teneur	% Daily Value % valeur quotidienne
Calories / Calories 260	21 %
Fat / Lipides 13 g	10 %
Saturated / saturés 2 g + Trans / trans 0 g	0 %
Cholesterol / Cholestérol 0 mg	0 %
Sodium / Sodium 370 mg	16 %
Carbohydrate / Glucides 31 g	10 %
Fibre / Fibres 2 g	9 %
Sugars / Sucres 1 g	
Protein / Protéines 3 g	
Vitamin A / Vitamine A	0 %
Vitamin C / Vitamine C	0 %
Calcium / Calcium	6 %
Iron / Fer	4 %

INGREDIENTS: SELECTED CORN, VEGETABLE OIL, SEASONING (CORN MALTODEXTRIN, SALT, CHEDDAR CHEESE, WHEY, MONOSODIUM GLUTAMATE, BUTTERMILK, ROMANO CHEESE, WHEY PROTEIN CONCENTRATE, ONION POWDER, VEGETABLE OIL, CORN FLOUR, NATURAL AND ARTIFICIAL FLAVOUR, DEXTROSE, TOMATO POWDER, LACTOSE, SPICES, COLOUR, LACTIC ACID, CITRIC ACID, SUGAR, GARLIC POWDER, SKIM MILK, RED AND GREEN BELL PEPPER POWDER, DISODIUM INOSINATE, DISODIUM GUANYLATE, MODIFIED CORN STARCH), CALCIUM HYDROXIDE.
CONTAINS MILK INGREDIENTS.

Roulette*



ATTENTION
SOME CHIPS VERY HOT
CERTAINES CHIPS SONT VRAIMENT ÉPICÉES

235 g

Nutrition Facts Valeur nutritive

Per 21 chips (50 g)
pour 21 croustilles (50 g)

Amount
Teneur

Calories / Calories

Fat / Lipides 13 g

Saturated / saturés
+ Trans / trans 0 g

Cholesterol / Cholestérol 0

Sodium 370 mg
Carbohydrate / Glucides 3



Trans Fat
0 gras trans

Doritos

Roulette



ATTENTION
SOME CHIPS VERY HOT
CERTAINES CHIPS SONT VRAIMENT ÉPICES

TORTILLA CHIPS
CHIPS TORTILLA AROMATISÉES

“Hot or Not”



“Hot or Not”

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



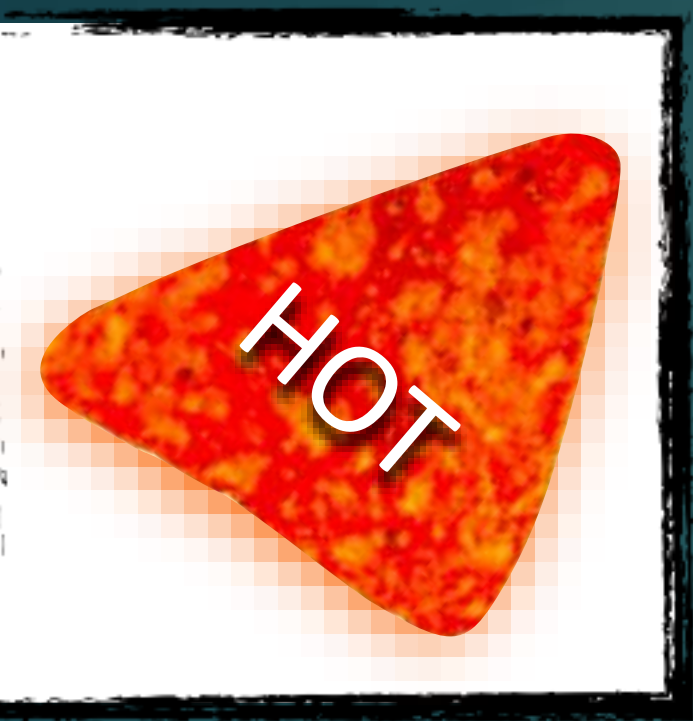
“Hot or Not”

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



“Hot or Not”

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



$$\frac{1}{6}$$

“Hot or Not”

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



1 ← “Hot” Chips

6

“Hot or Not”

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?

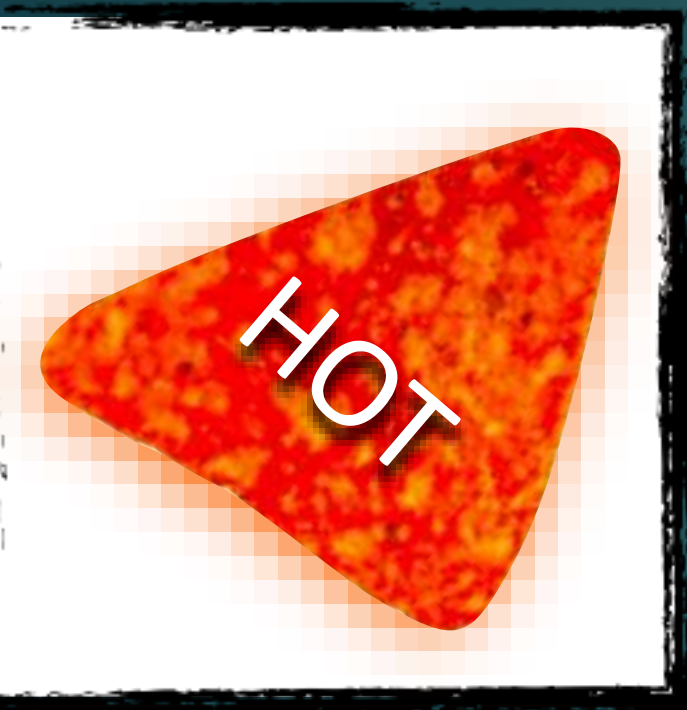


1 ← “Hot” Chips

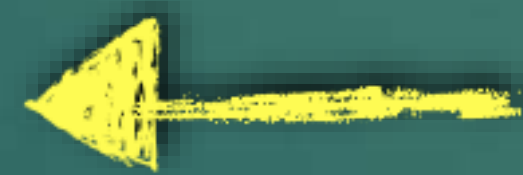
6 ← “Not Hot” Chips

“Hot or Not”

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



1



“Hot” Chips

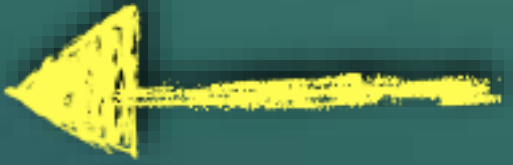
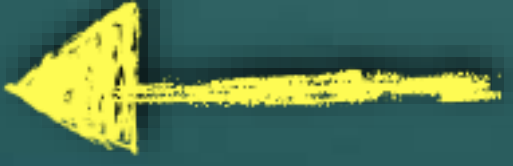
6



“Not Hot” Chips

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



1  "Hot" Chips
6  "Not Hot" Chips

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



1 ← "Hot" Chips
6 ← "Not Hot" Chips

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



"Hot"

"Not Hot"

1 ← "Hot" Chips
6 ← "Not Hot" Chips

What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



0 1 2 3 4 5 6 7

"Hot"

"Not Hot"

1 ← "Hot" Chips

6 ← "Not Hot" Chips



1 ← "Hot" Chips
6 ← "Not Hot" Chips



0 1 2 3 4 5 6 7



Part-Part Relationship



What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



"Hot"

"Not Hot"

1 ← "Hot" Chips

6 ← "Not Hot" Chips

Baking Brownies



Baking Brownies

How can 4 friends share 6 brownies fairly?



Baking Brownies

How can 4 friends share 6 brownies fairly?



Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

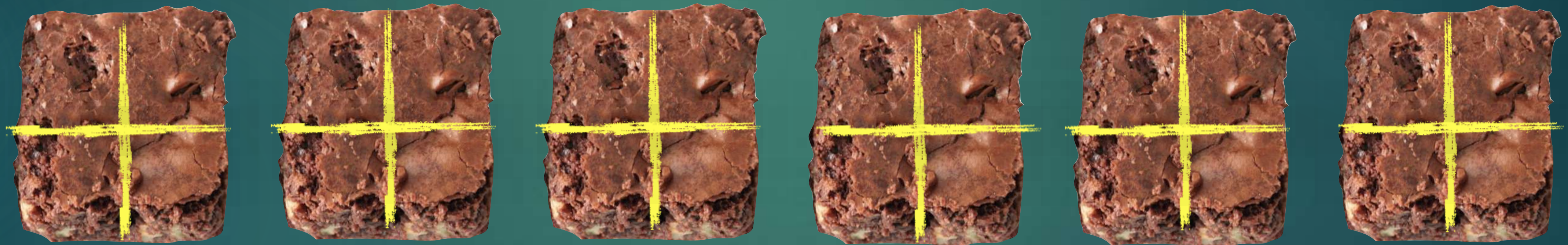


Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partitioning Brownies into Fourths

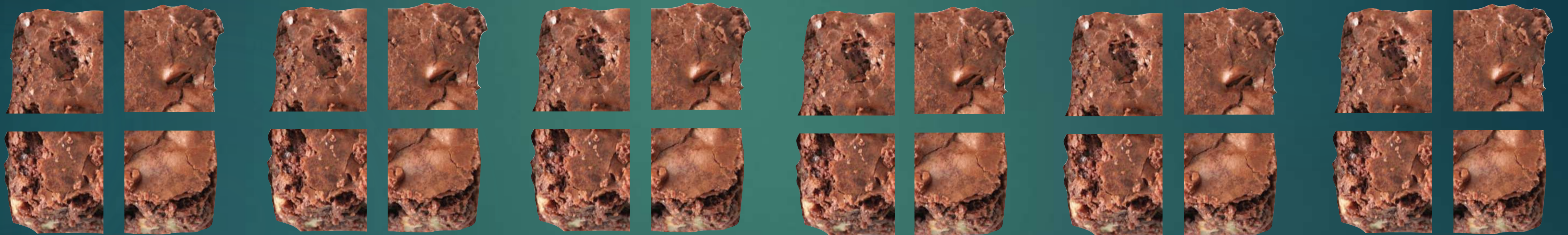


Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partitioning Brownies into Fourths



6 brownies

4 people

Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partitioning Brownies into Fourths



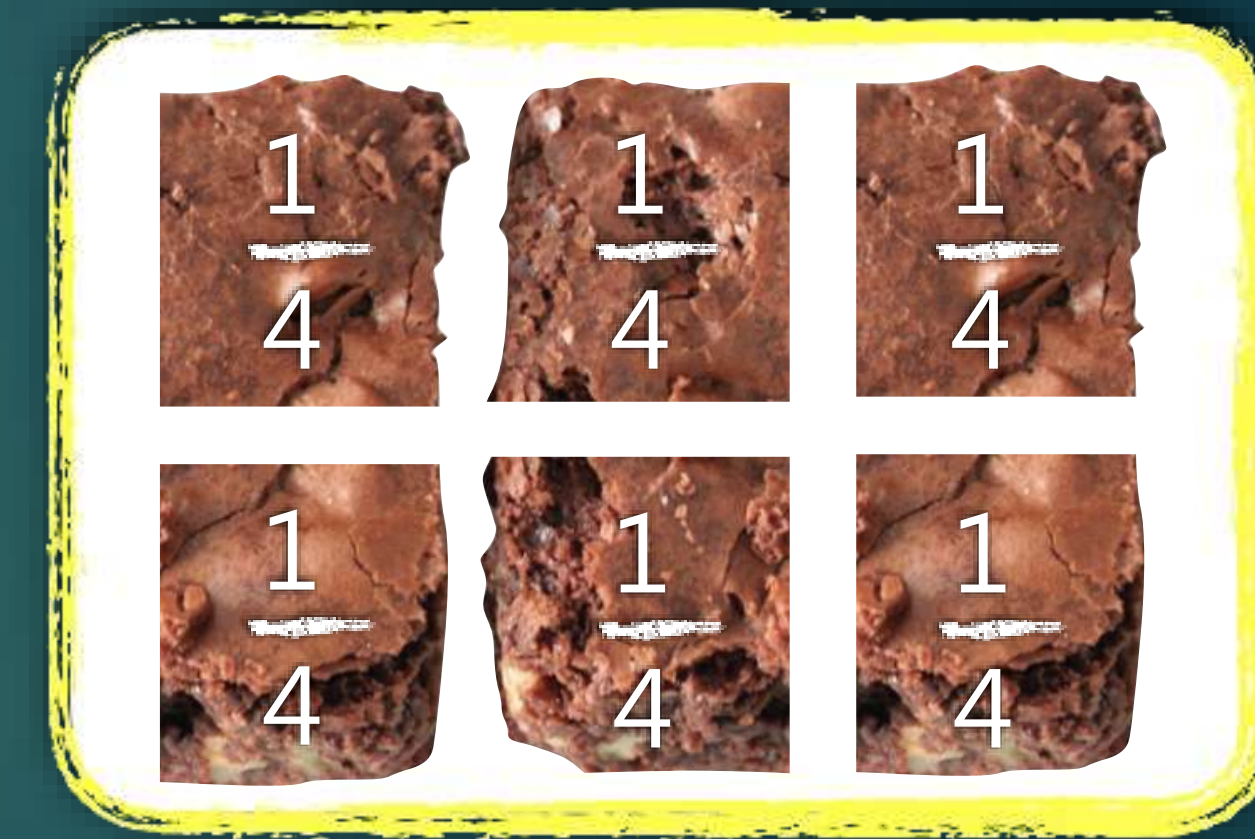
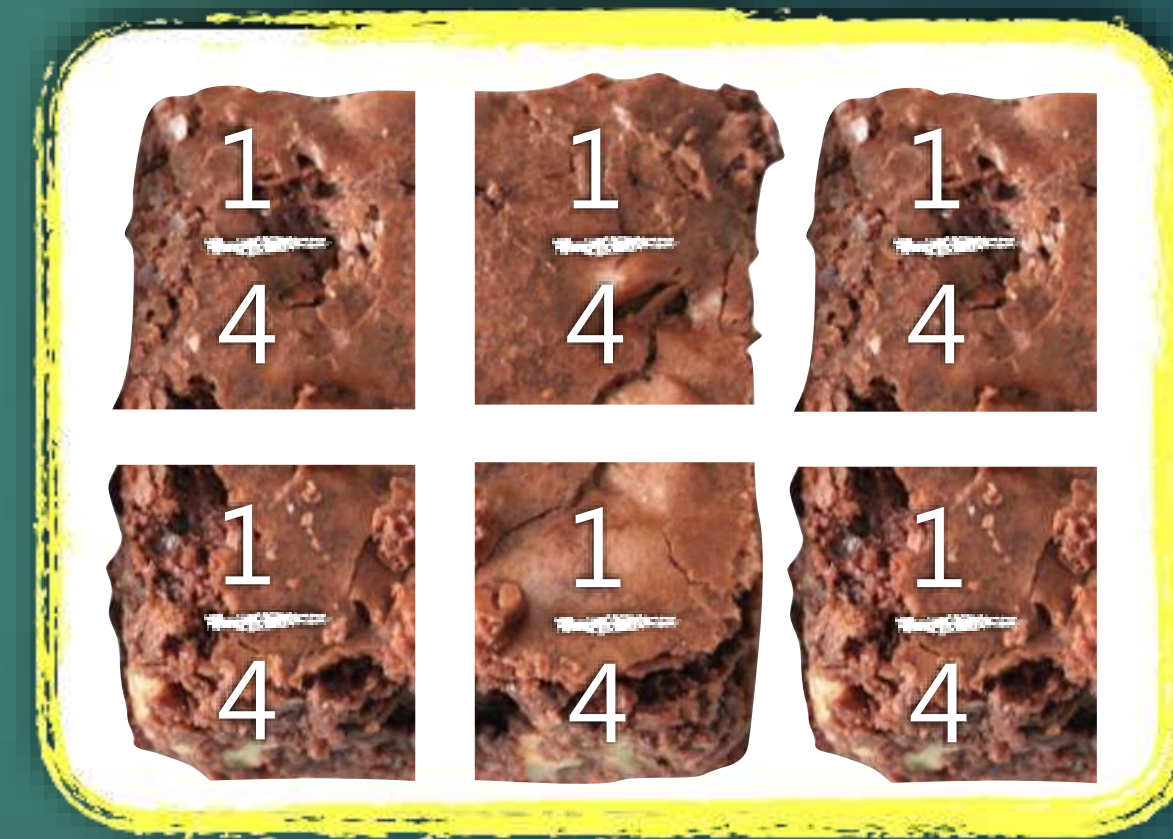
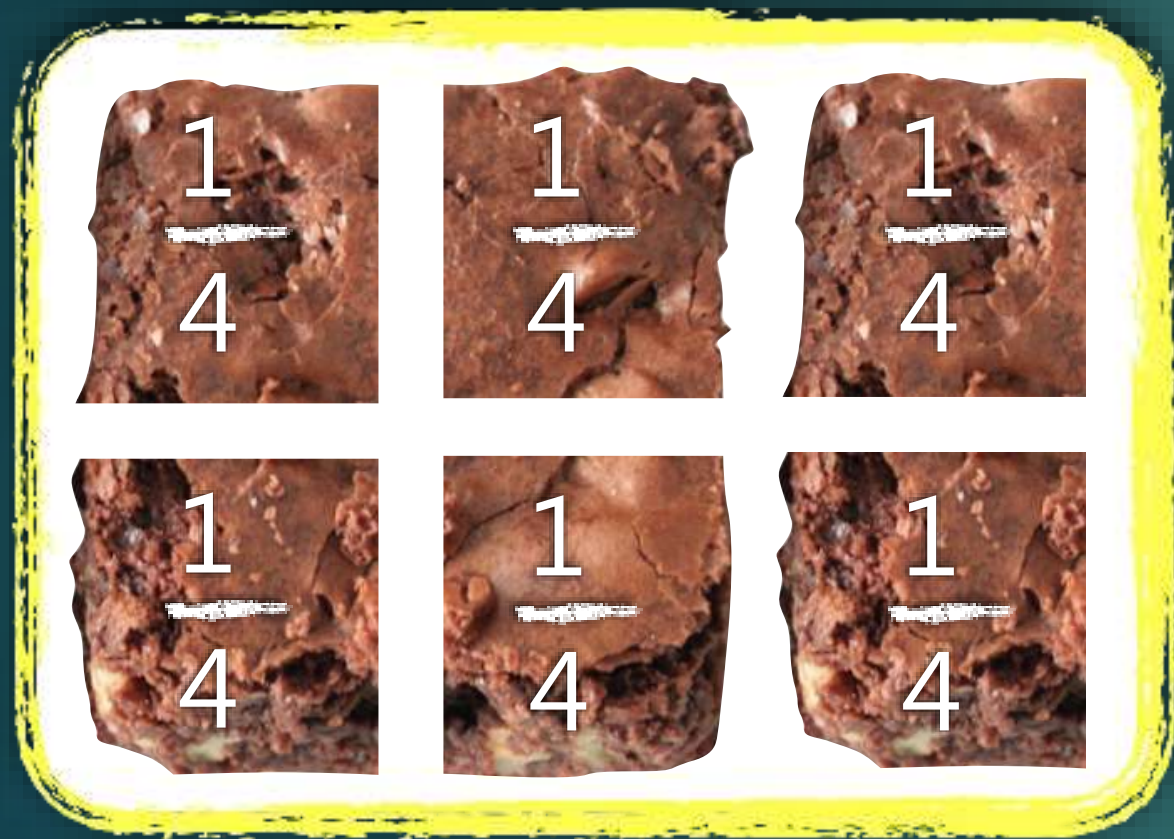
$$\frac{6 \text{ brownies}}{4 \text{ people}} = \frac{6}{4}$$

Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partitioning Brownies into Fourths



$$\frac{6 \text{ brownies}}{4 \text{ people}} = \frac{6}{4}$$

Each person would get 6 one-fourth pieces of brownies.

Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Sharing full brownies equally

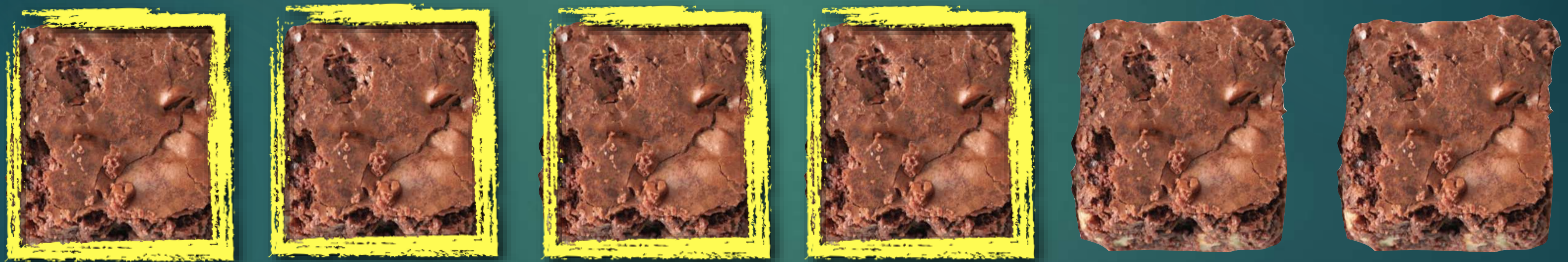


Baking Brownies

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Multiple Representations:

Sharing full brownies equally

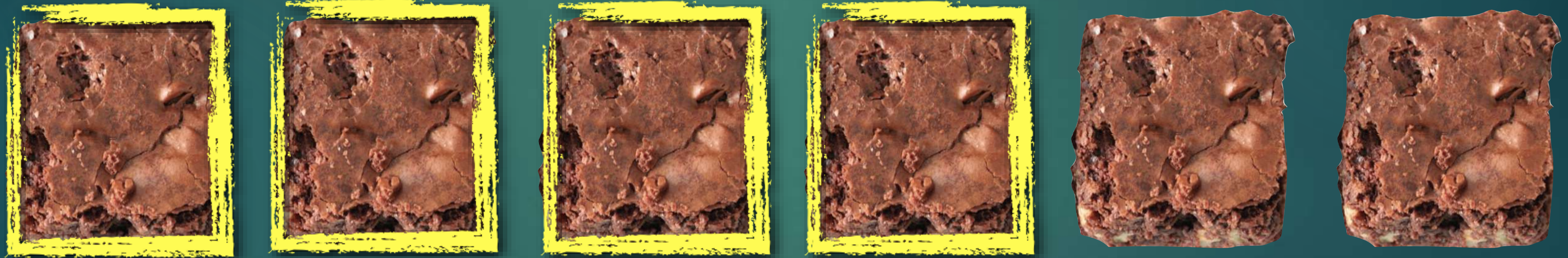


Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Sharing full brownies equally, then partitioning the remainder.



Baking Brownies

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Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Sharing full brownies equally, then partitioning the remainder.



$$\frac{6 \text{ brownies}}{4 \text{ people}} = 1 \frac{1}{2}$$

Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Sharing full brownies equally, then partitioning the remainder.



$$\frac{6 \text{ brownies}}{4 \text{ people}} = 1 \frac{1}{2}$$

Each person would get 1 full and 1 half piece of brownies.

Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partition the brownies into four equal-size portions

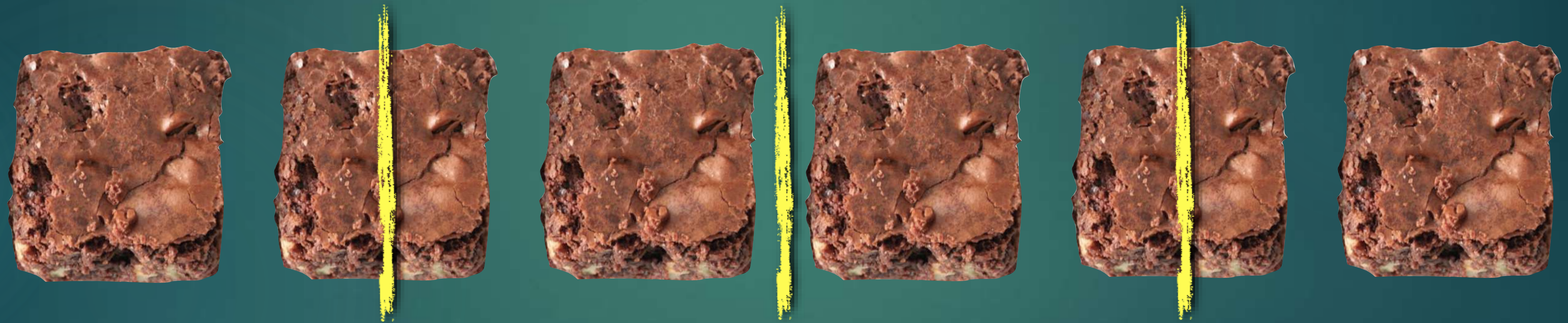


Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partition the brownies into four equal-size portions



Baking Brownies

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Multiple Representations:

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$$\frac{6 \text{ brownies}}{4 \text{ people}} = 1 \frac{1}{2}$$

Baking Brownies

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partition the brownies into four equal-size portions



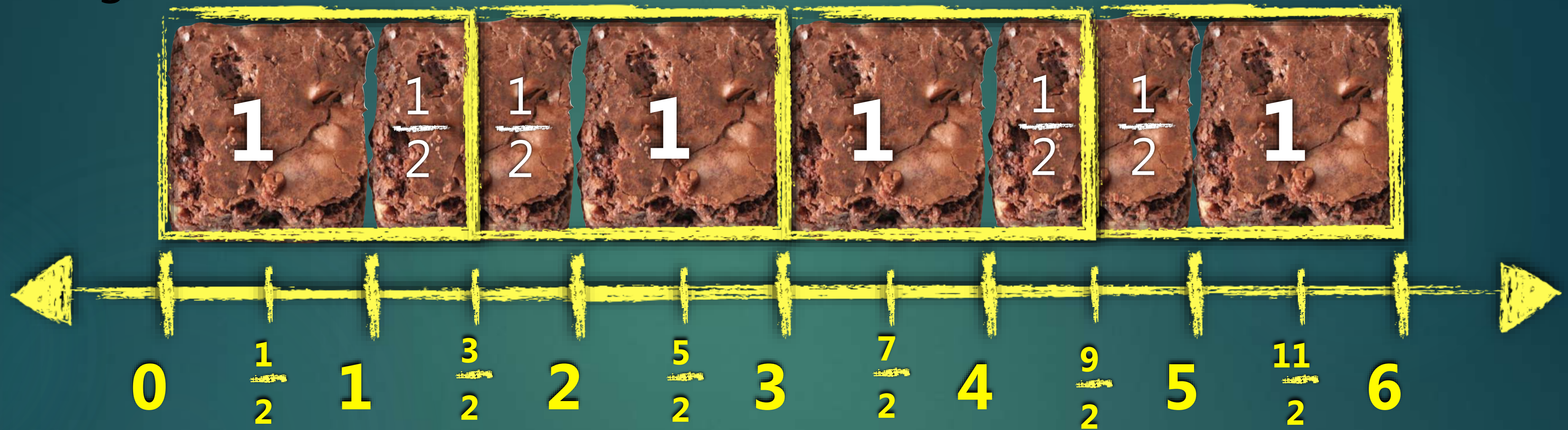
$$\frac{6 \text{ brownies}}{4 \text{ people}} = 1 \frac{1}{2}$$

Each person would get 1 full and 1 half piece of brownies.

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Using a number line



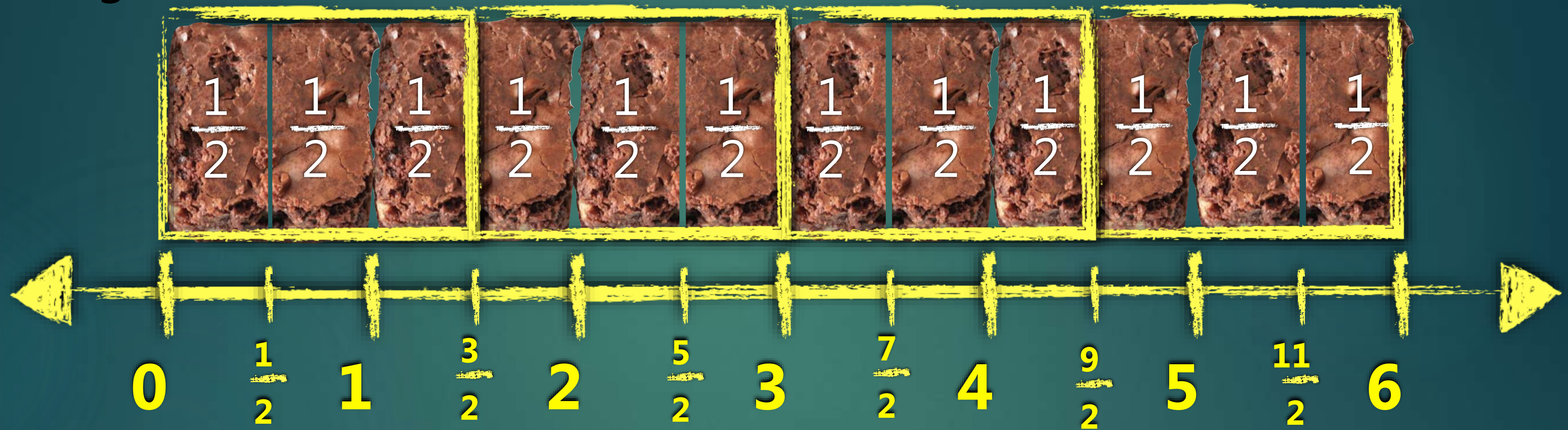
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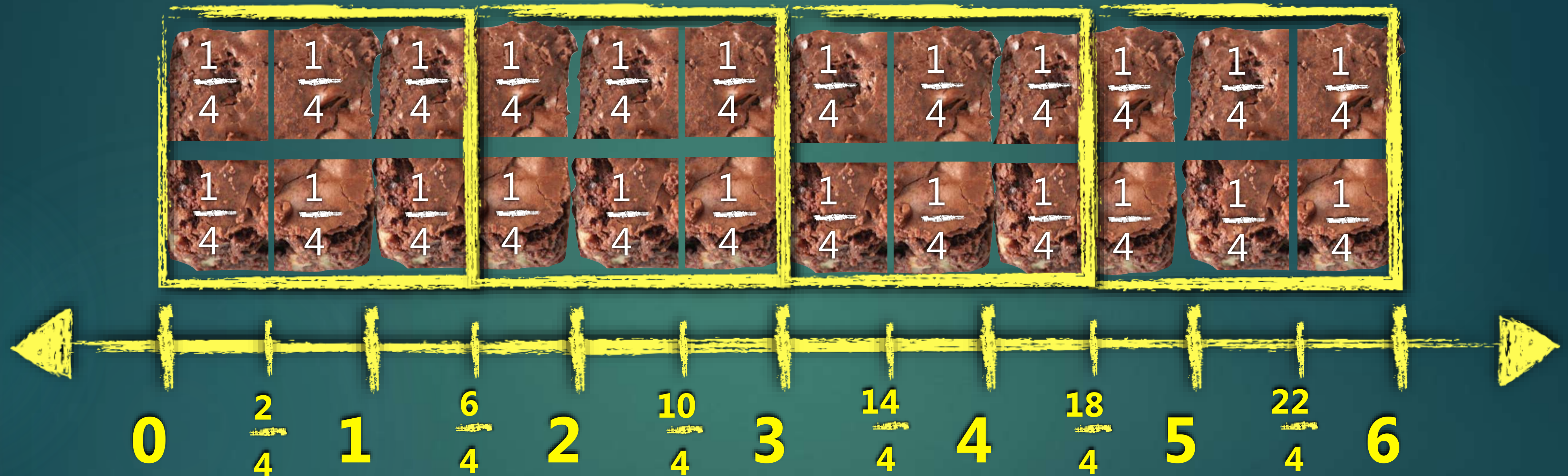
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How can 4 friends share 6 brownies fairly?

Multiple Representations:

Using a number line



$$\frac{6 \text{ brownies}}{4 \text{ people}}$$

$$= 1 \frac{1}{2}$$

Each person would get 1 full and 1 half piece of brownies.

Fraction as Quotient

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partition the brownies into four equal-size portions



6 brownies

4 people

$$= 1 \frac{1}{2}$$

Each person would get 1 full and 1 half piece of brownies.

How Many Rolos?



How Many Rolos?

There are 7 pieces in every roll of Rolo chocolate.



How Many Rolos?



There are 7 pieces in every full roll of Rolo chocolate.

Two partially eaten rolls are found in a drawer; one with 5 pieces and the other with 4 pieces. How many full rolls of Rolo are there?

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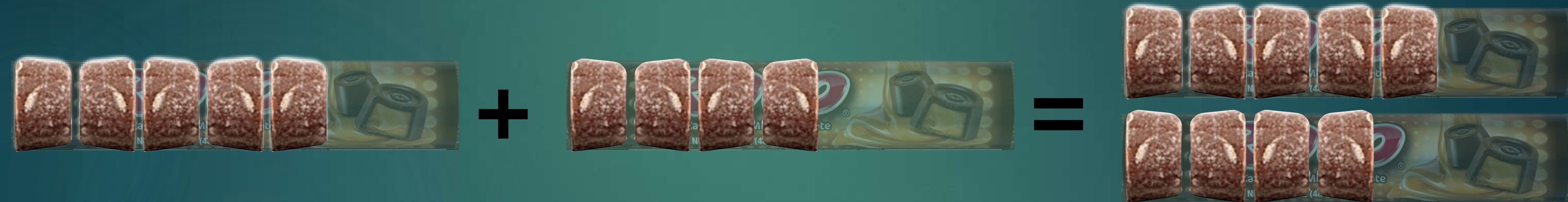
+



How Many Rolos?

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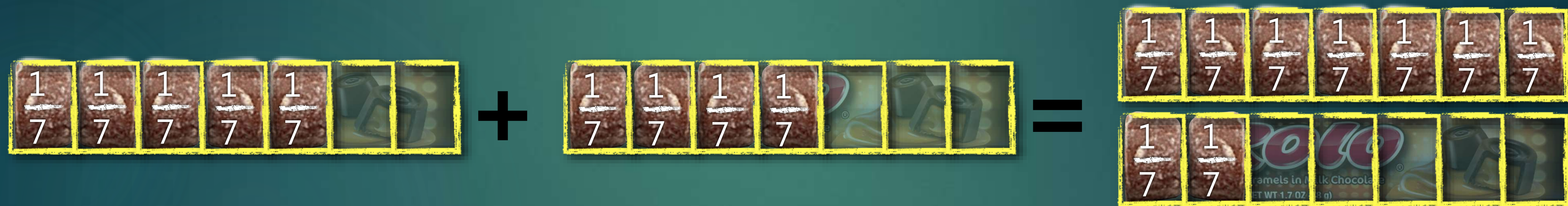
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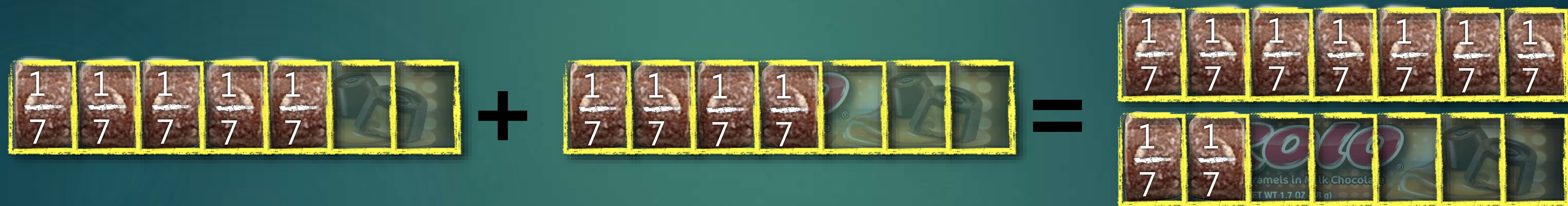
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How Many Rolos?

There are 7 pieces in every full roll of Rolo chocolate.

Two partially eaten rolls are found in a drawer; one with 5 pieces and the other with 4 pieces. How many full rolls of Rolo are there?



5 pieces

7 pieces

+

4 pieces

7 pieces

=

9 pieces

7 pieces

There are 7 pieces in every full roll of Rolo chocolate.

Two partially eaten rolls are found in a drawer; one with 5 pieces and the other with 4 pieces. How many full rolls of Rolo are there?



5 pieces

+

4 pieces

=

9 pieces

7 pieces

7 pieces

7 pieces

There are 7 pieces in every full roll of Rolo chocolate.

Two partially eaten rolls are found in a drawer; one with 5 pieces and the other with 4 pieces. How many full rolls of Rolo are there?



$$\frac{5 \text{ pieces}}{7 \text{ pieces}}$$

+

$$\frac{4 \text{ pieces}}{7 \text{ pieces}}$$

=

$$\frac{9 \text{ pieces}}{7 \text{ pieces}}$$

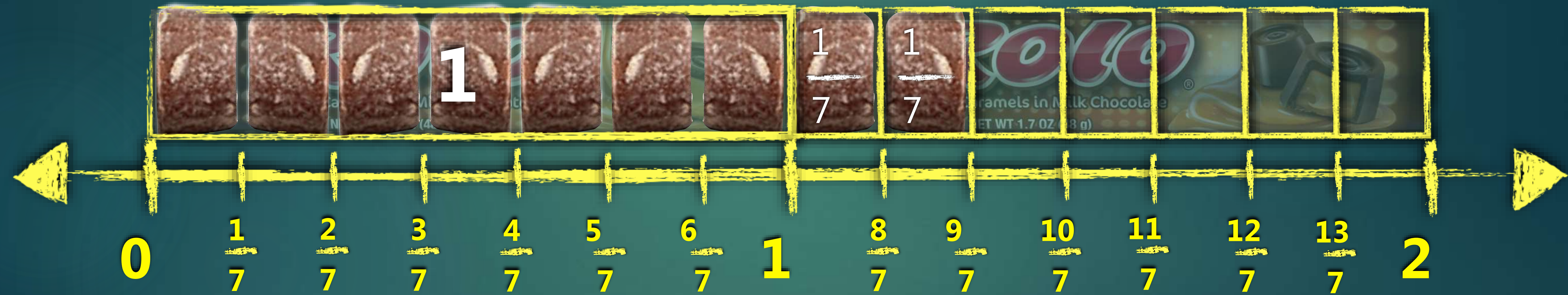
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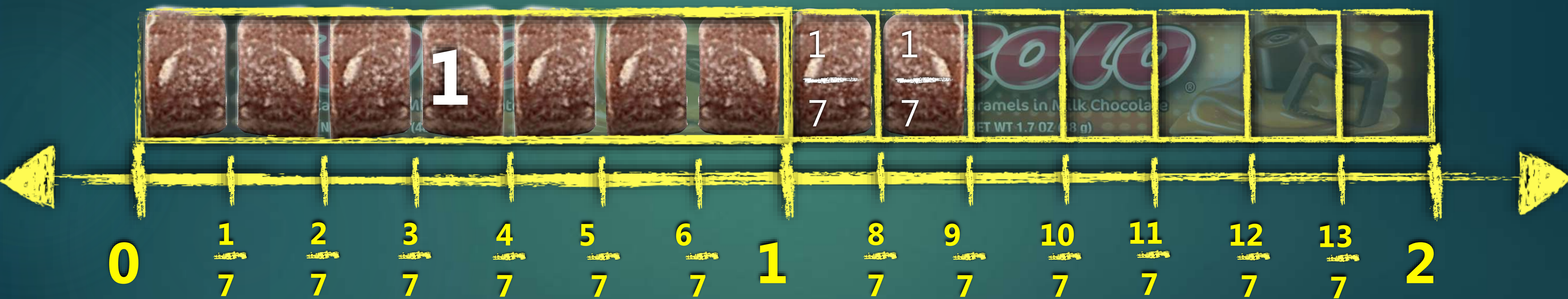
$$\frac{5}{7} + \frac{4}{7} = 1 \frac{2}{7}$$

There is one full roll of Rolos and an additional 2 pieces out of a roll of 7.

Fractions As Operators

There are 7 pieces in every full roll of Rolo chocolate.

Two partially eaten rolls are found in a drawer; one with 5 pieces and the other with 4 pieces. How many full rolls of Rolo are there?

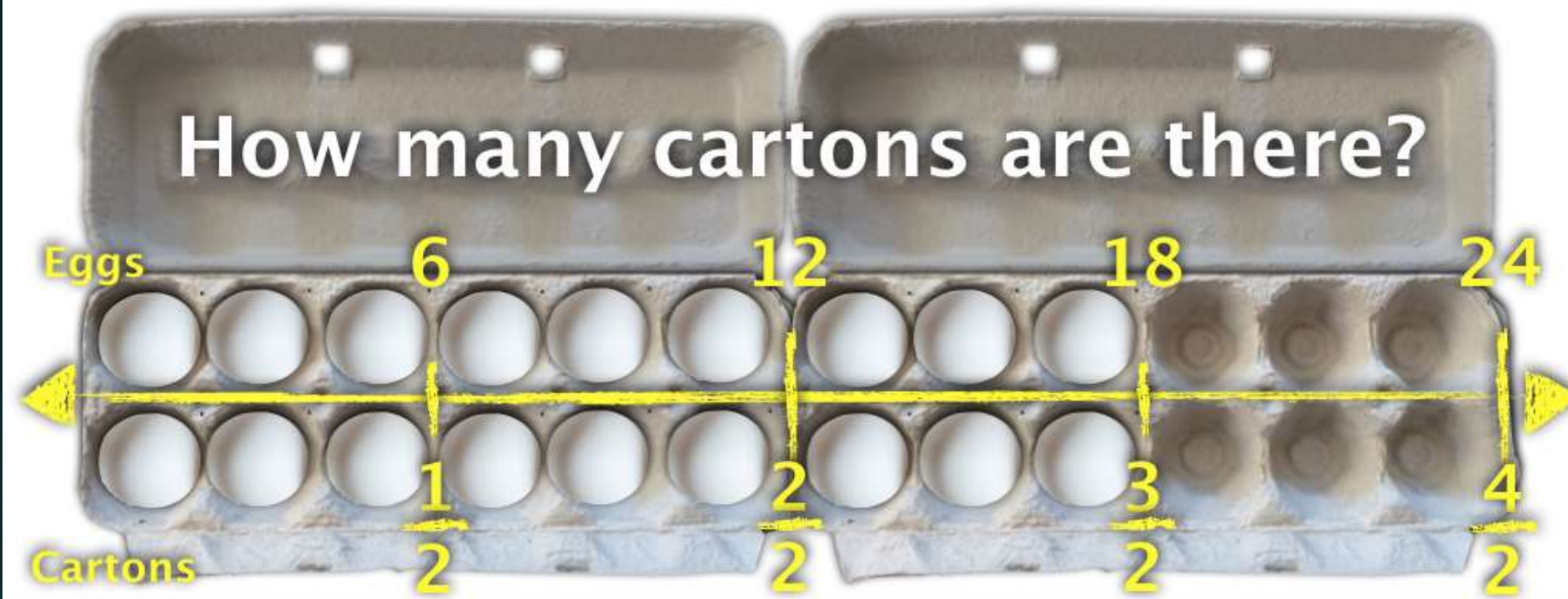


$$\frac{5}{7} + \frac{4}{7} = 1\frac{2}{7}$$

There is one full roll of Rolos and an additional 2 pieces out of a roll of 7.

Fraction Constructs

Part-Whole Relationship



Fraction as Quotient

How can 4 friends share 6 brownies fairly?

Multiple Representations:

Partition the brownies into four equal-size portions

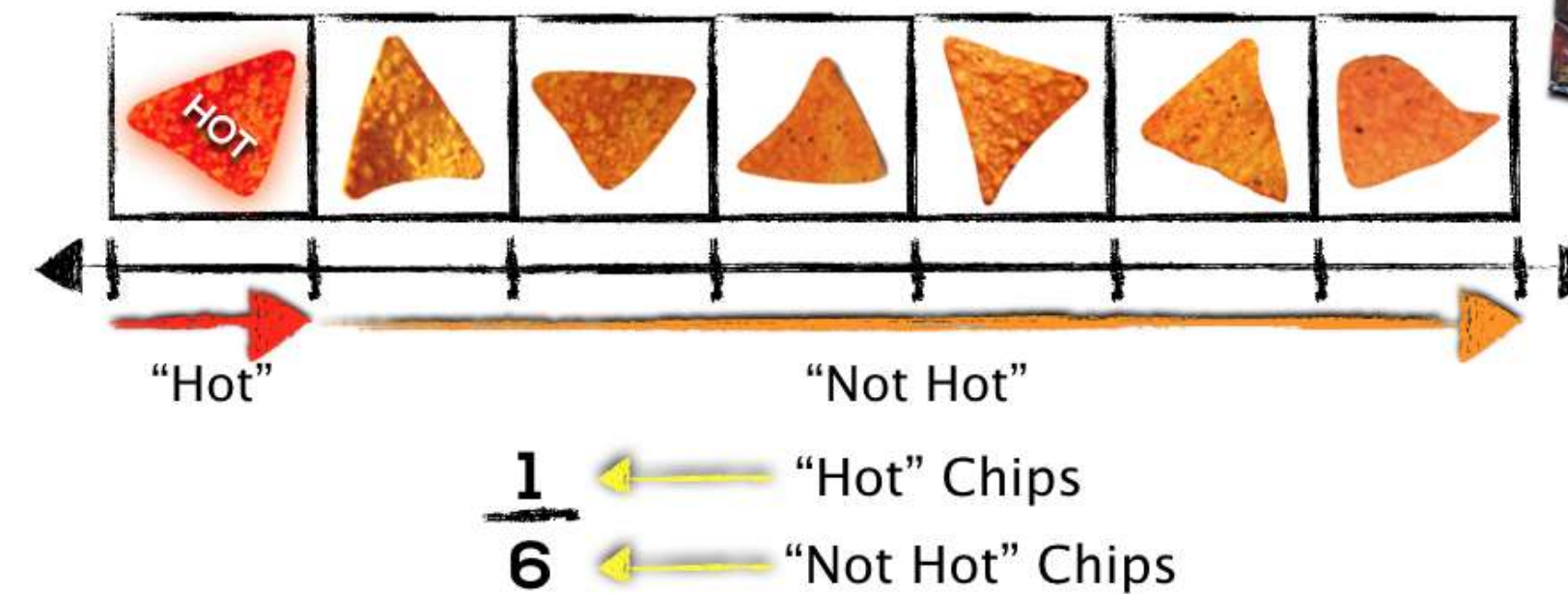


$$\frac{6 \text{ brownies}}{4 \text{ people}} = 1 \frac{1}{2}$$

Each person would get 1 full and 1 half piece of brownies.

Part-Part Relationship

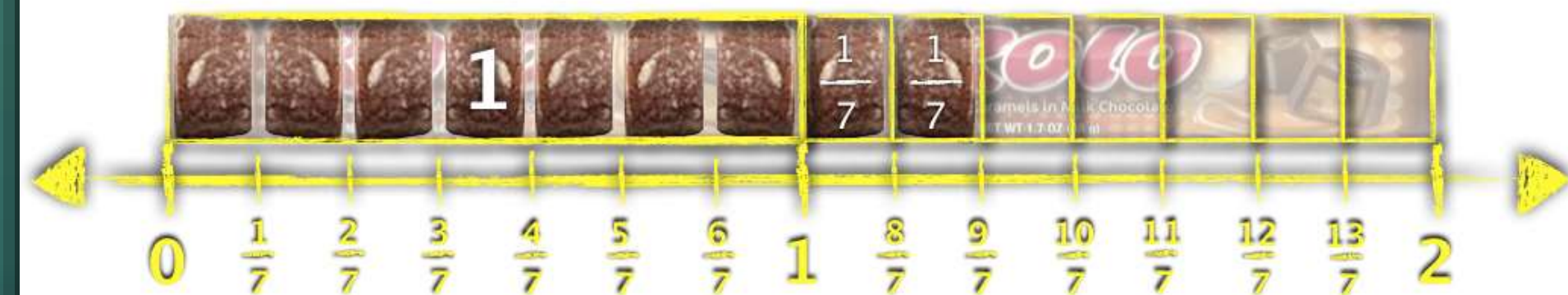
What fraction of hot chips to not hot chips are there in a bag of Doritos Roulette?



Fractions As Operators

There are 7 pieces in every full roll of Rolo chocolate.

Two partially eaten rolls are found in a drawer; one with 5 pieces and the other with 4 pieces. How many full rolls of Rolo are there?



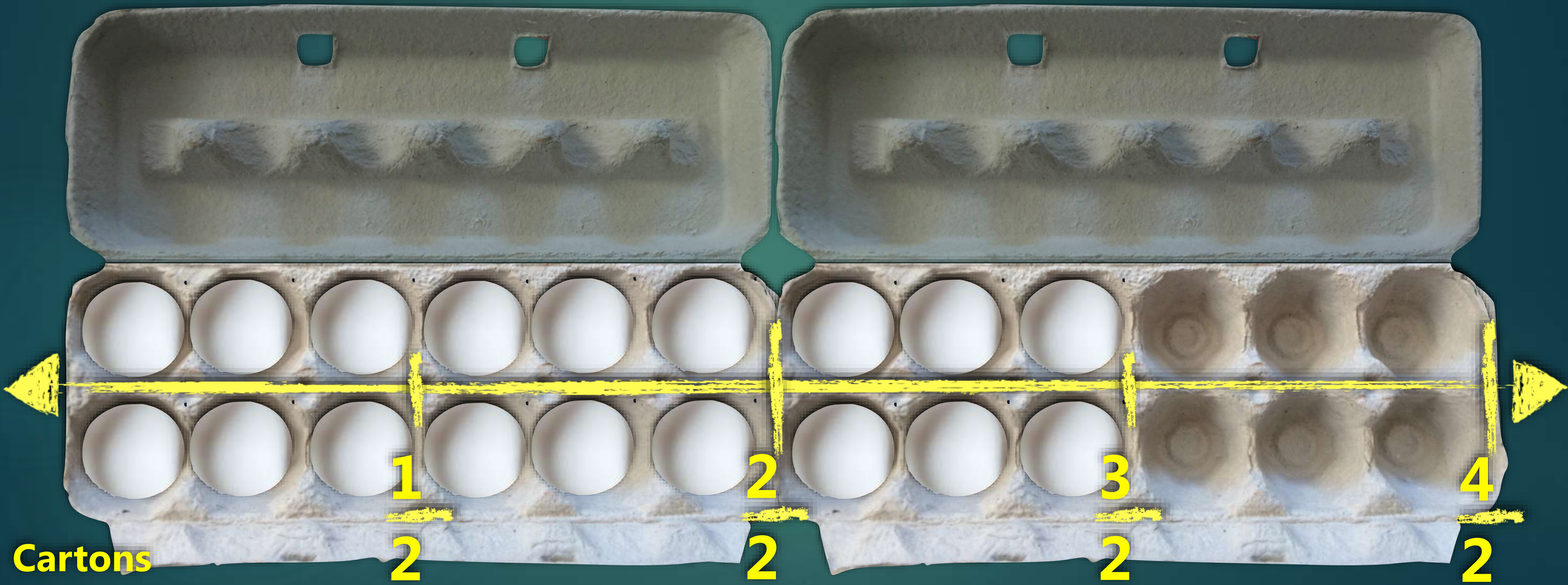
$$\frac{5}{7} + \frac{4}{7} = 1 \frac{2}{7}$$

There is one full roll of Rolos and an additional 2 pieces out of a roll of 7.



Four Fraction Models

Four Fraction Models



Cartons

1
2

2
2

3
2

4
2

Four Fraction Models

Linear Model

Cartons



DOUBLE CLOTHESLINE



DOUBLE CLOTHESLINE

0.2

DOUBLE CLOTHESLINE

0.2

DOUBLE CLOTHESLINE

0.2

? %

DOUBLE CLOTHESLINE

0.2

? %

DOUBLE CLOTHESLINE



0.2

0

? %

0 %

DOUBLE CLOTHESLINE



0

0.2

0 %

? %

DOUBLE CLOTHESLINE



0

0.2

1

0 %

? %

DOUBLE CLOTHESLINE



0

0.2

1

0 %

? %

DOUBLE CLOTHESLINE



0

1

0.2

0 %

? %

DOUBLE CLOTHESLINE



0

0.2

1

0 %

? %

DOUBLE CLOTHESLINE



0

0.2

1

0 %

? %

DOUBLE CLOTHESLINE



0

0.2

1

0 %

20 %

100 %

DOUBLE CLOTHESLINE



0

0.2

0.4

1

0 %

20 %

40 %

100 %

DOUBLE CLOTHESLINE



0

0.2

0.4

1

0 %

20 %

40 %

100 %

DOUBLE CLOTHESLINE



0

0.2

0.4

0.6

0.8

1

0 %

20 %

40 %

60 %

80 %

100 %

DOUBLE CLOTHESLINE



0

0.2

0.4

0.6

0.8

1

0 %

20 %

40 %

60 %

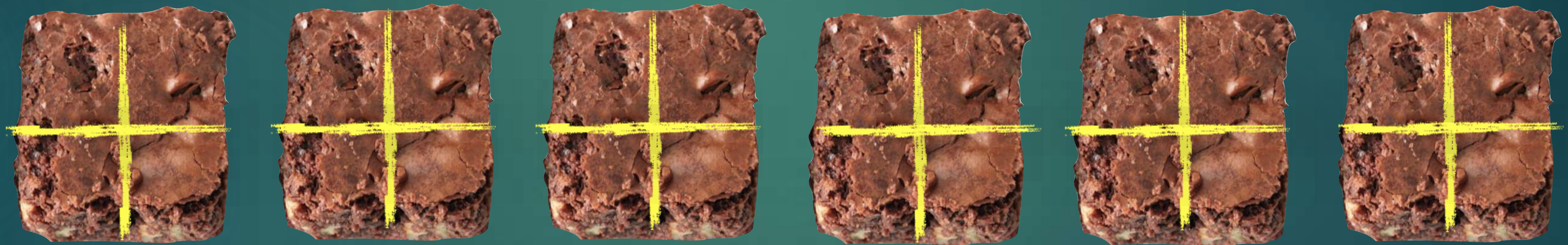
80 %

100 %

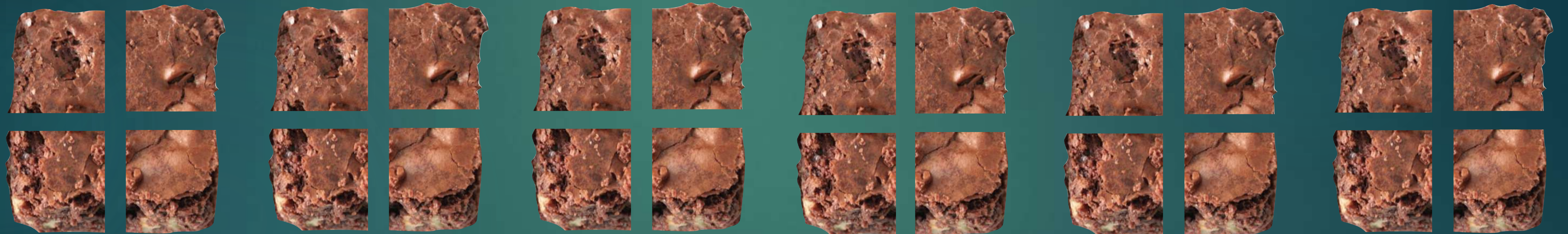
Four Fraction Models



Four Fraction Models

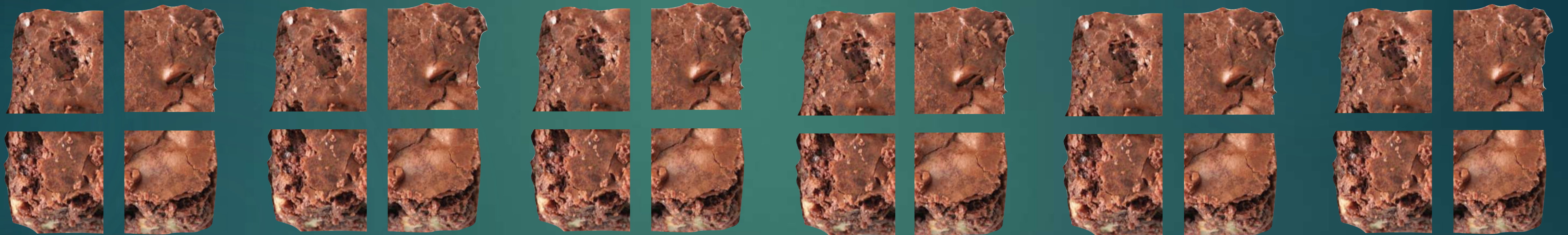


Four Fraction Models

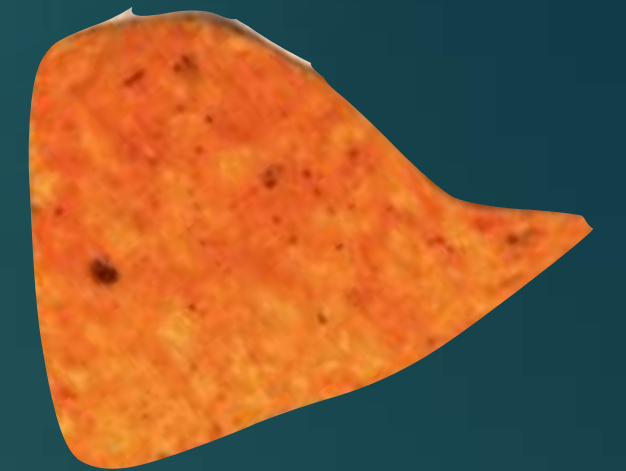
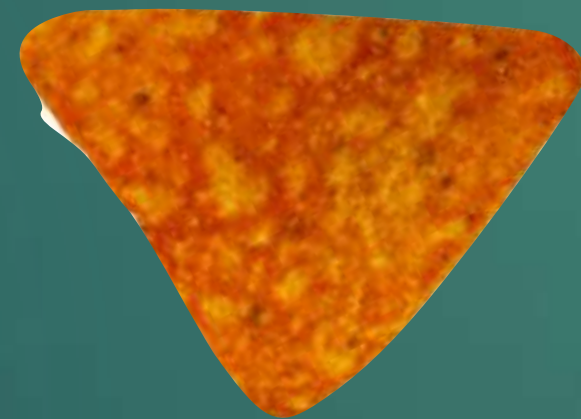


Four Fraction Models

Area Model

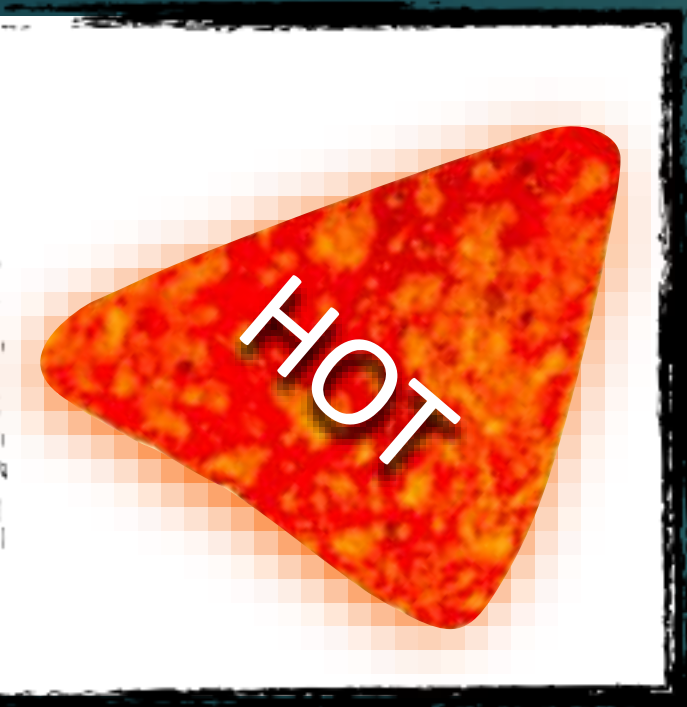


Four Fraction Models



Four Fraction Models

Set Model



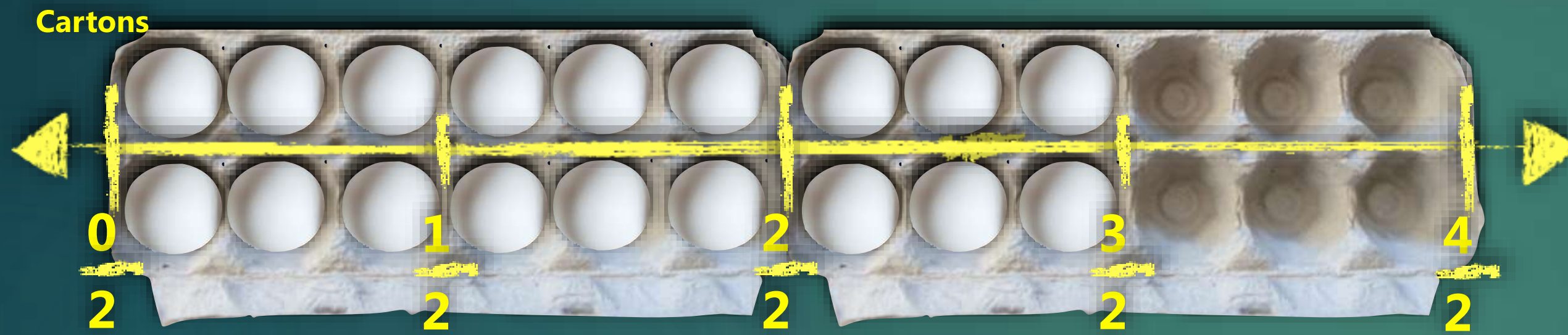
Four Fraction Models

Volume Model

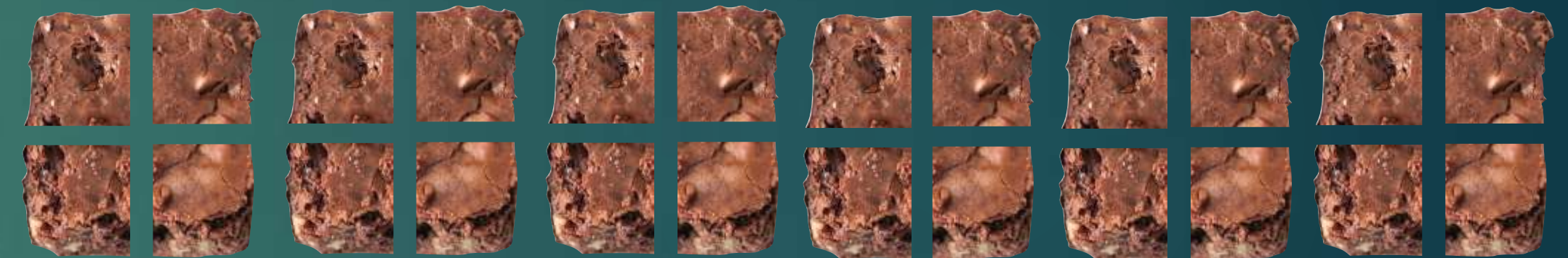


Four Fraction Models

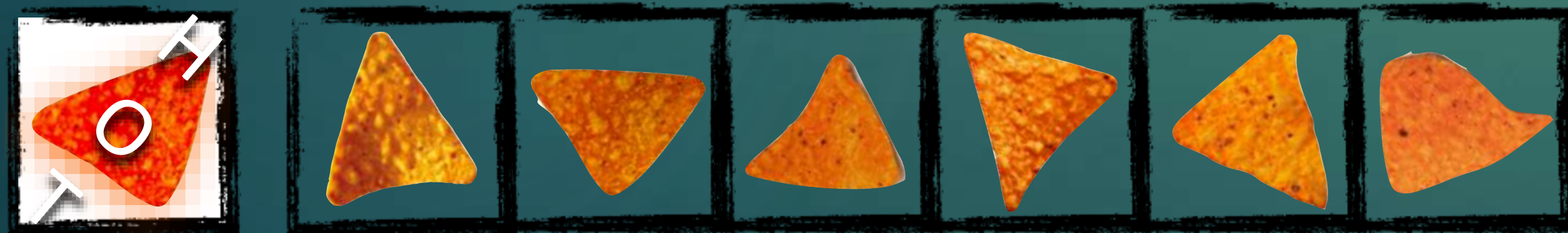
Linear Model



Area Model

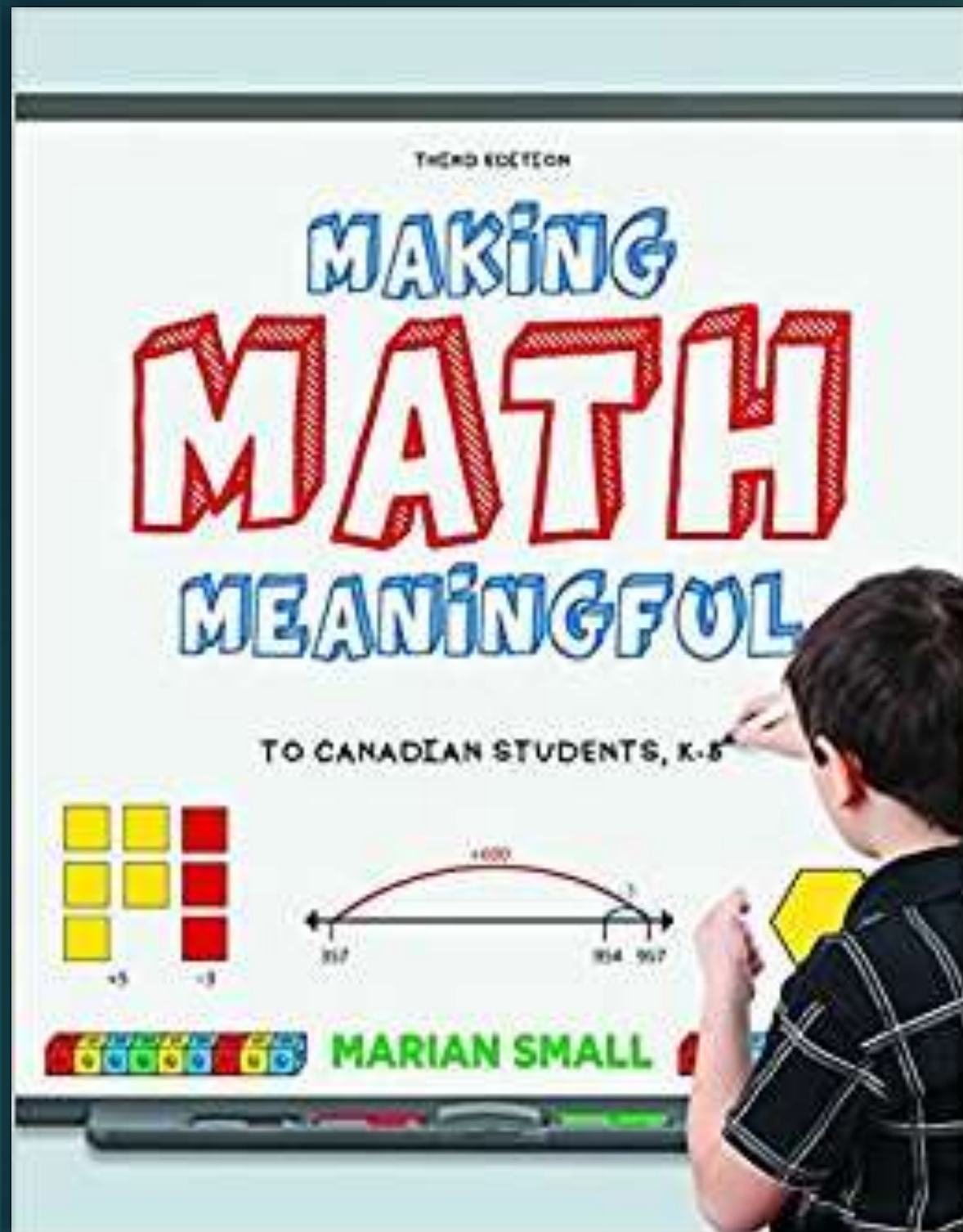


Set Model



Volume Model

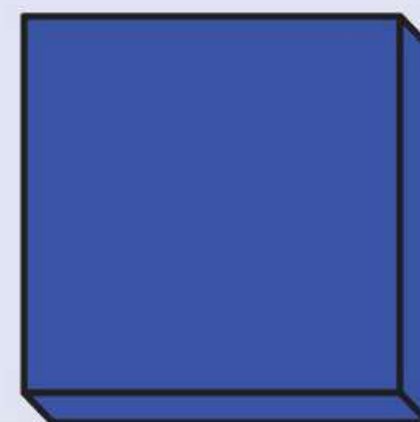
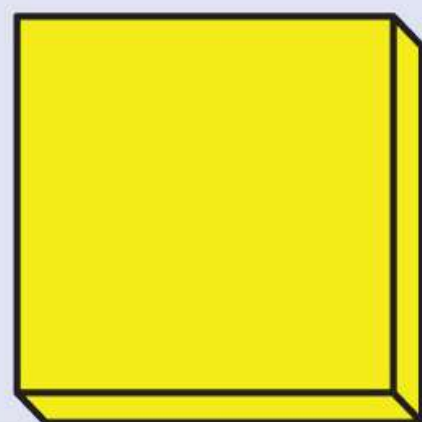
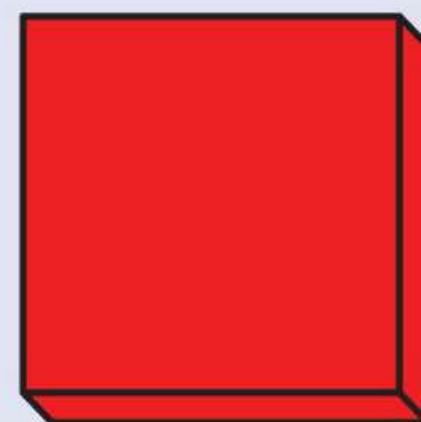




Create a design where there are

- twice as many yellow squares as red ones and
- twice as many blue squares as yellow ones.

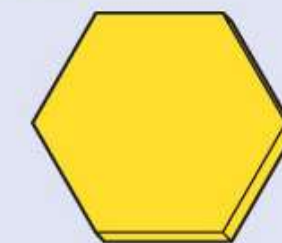
What fraction of the area is yellow?



Create a design where there are

- twice as many yellow pattern blocks as red ones and
- twice as many blue pattern blocks as yellow ones.

What fraction of the area is yellow?

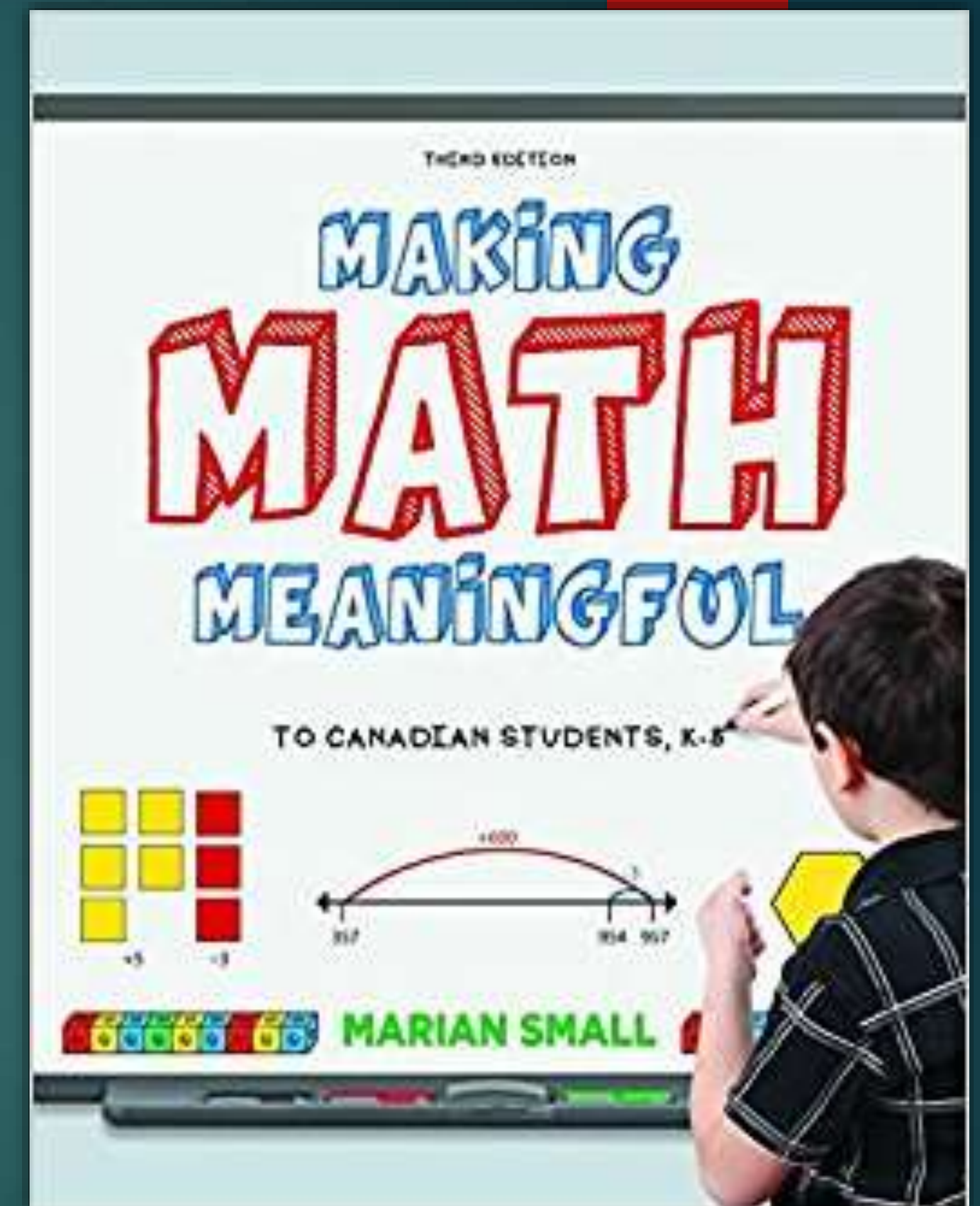


ACTIVITY 12.4

Display circle graphs like this one to students. Ask questions related to fractions about them. For example,

- *About what fraction of the school population is in each grade? How do you know?*
- *What is the probability that a student chosen at random is a Grade 1 student?*
- *How might the graph change if about $\frac{1}{4}$ of the students were in kindergarten?*

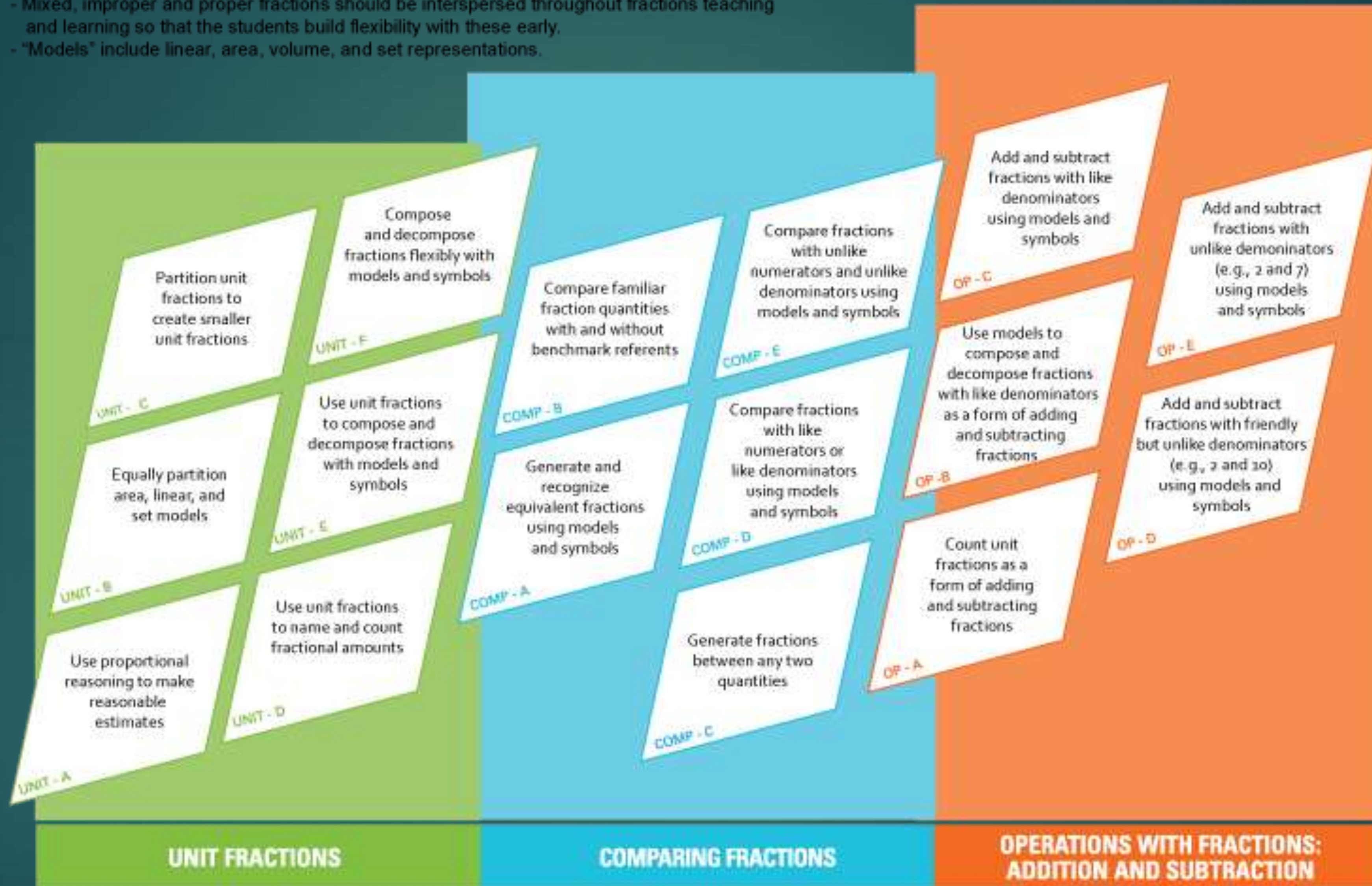
Students in K to Grade 3



Fractions Learning Pathways

Please Note:

- Mixed, improper and proper fractions should be interspersed throughout fractions teaching and learning so that the students build flexibility with these early.
- "Models" include linear, area, volume, and set representations.



Partition unit fractions to create smaller unit fractions

UNIT - C

Compose and decompose fractions flexibly with models and symbols

UNIT - F

Equally partition area, linear, and set models

UNIT - B

Use unit fractions to compose and decompose fractions with models and symbols

UNIT - E

Use proportional reasoning to make reasonable estimates

UNIT - A

Use unit fractions to name and count fractional amounts

UNIT - D

UNIT FRACTIONS

Compare familiar fraction quantities with and without benchmark referents

COMP - B

Generate and recognize equivalent fractions using models and symbols

COMP - A

Generate fractions between any two quantities

COMP - C

COMPARING FRACTIONS

Compare fractions with unlike numerators and unlike denominators using models and symbols

COMP - E

Compare fractions with like numerators or like denominators using models and symbols

COMP - D

Count unit fractions as a form of adding and subtracting fractions

OP - A

Use models to compose and decompose fractions with like denominators as a form of adding and subtracting fractions

OP - B

Add and subtract fractions with like denominators using models and symbols

OP - C

Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols

OP - D

Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols

OP - E

OPERATIONS WITH FRACTIONS: ADDITION AND SUBTRACTION



Unit
A

Unit Fractions

Use proportional reasoning to make reasonable estimates

Understanding proportional reasoning requires students to consider a number or quantity in relative terms. With fractions, students must consider the fraction in context, such as one half of a whole figure. Students use proportional reasoning to partition a whole into unit fractions.

BACKGROUND

A unit fraction is the base unit of any fraction and always has a numerator of 1; for example, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{23}$ are all unit fractions. Every

TASKS

Walk the Line

Students actively equi-partition a number line using different fractional units (e.g., halves, fifths) as they place mixed and improper fractions. Students will enjoy walking, jumping or using every day classroom items as a method of kinaesthetically partitioning a

Walk the Line



Description

Students actively equi-partition a number line using different fractional units (e.g., halves, fifths) as they place mixed and improper fractions. Students will enjoy walking, jumping or using every day classroom items as a method of kinaesthetically partitioning a number line on the floor. This task becomes increasingly complex based upon the sets of fractions used.

Mathematics

Accurately placing fractions on a number line involves significant spatial reasoning and the use of a large number line allows students to gesture and walk to communicate their spatial reasoning. Research shows that the number line is a powerful model for representing fractions that supports a deeper understanding of fraction as number (as opposed to a circle model). Unit amounts are purposefully scaffolded to allow students to use their knowledge of benchmark fractions (e.g., $\frac{1}{5}$) to place other fractions (e.g., $\frac{6}{5}$).

Curriculum Connections

Students will:

- understand a fraction as a number on the number line;
- represent and compare fractions;
- accurately place fractions on a number line by reasoning about their relative size.

Instructional sequence

Explore the Progression



Explore the Fraction Learning Pathway with a partner.

Where do you feel your grade level curriculum aligns with the Pathway?

Pick an Activity



Select a topic from the Pathway.

Read the topic and explore the suggested activity.

Be ready to share out your thinking to the group.

Making Sense Series

The Progression of Fractions
Meaning, Equivalence, & Comparison

[gfletchyfractions](https://www.gfletchy.com)

created by Graham Fletcher

 @gfletchy

www.gfletchy.com

Progression of Fractions

